String Cosmology

Modern String Theory Concepts
from the Big Bang to Cosmic Structure

Edited by
Johanna Erdmenger

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Contents

Preface

Cosmology has made huge steps forward over the last twenty-five years, both through new observations as well as through phenomenological models. Important cosmological parameters have been measured with unprecedented accuracy. For instance, measurements of the cosmic microwave background have severely constrained the possible models describing the early universe.

At the same time, string theory has progressed as a most promising candidate for a quantum theory of gravity. Simultaneously, it provides a unified framework for all four fundamental interactions, including the standard model of elementary particles in addition to gravity.

Nevertheless, important questions remain. On one hand, theorists aim for a microscopical understanding of the effective theories describing the early universe. Also, the physics close to the initial singularity of the universe remains to be understood. This requires a full quantum theory of gravity. On the other hand, new and forthcoming precision measurements, such as of the fluctuations in the cosmic microwave background, will provide possibilities for further detailed tests of theories describing the early universe.

Recently, string theory has taken up the challenge of deriving experimentally or observationally testable predictions. This applies in particular to cosmology, as the examples in this book show. This is in particular due to the fact that cosmology allows one to access very high-energy scales in the early universe. An important activity in recent years has been to obtain inflation, that is a period of accelerated exponential expansion in the early universe, within string-theoretical models. The forthcoming experiments may potentially discriminate between different classes of these models. For instance, some of these models predict new structures in the power spectrum of cosmic microwave background fluctuations in a very natural way. Moreover, recent new developments in string theory have shed new light on the possibility that microscopic superstrings created in the early universe could have been magnified to macroscopic size during the cosmic expansion, possibly leading to astronomically observable consequences. Also, string theory may be useful in gaining new understanding of the origin of the cosmological constant and of the nature of dark energy. It may also provide mechanisms by which primordial gravitational waves can be generated.

It should also be mentioned here that new experimental predictions from string theory are also emerging in relation to elementary particle physics. For example, cross-sections for reaction processes have been calculated under the assumption of a low string scale of about 1 TeV. These can be tested at the Large Hadron Collider (LHC) at the CERN laboratory in Geneva, whose experiments will begin to produce data soon. Also, the searches for supersymmetry may have consequences both for string theory and for cosmology. Furthermore, the correspondence between supergravity in Anti-de Sitter spaces and conformal field theories (AdS/CFT correspondence) and its generalizations have provided new relations between string theory and strongly coupled gauge theories, in particular gauge theories relevant for heavy-ion physics and for the quark-gluon plasma, which is expected to play a role in structure formation in the early universe. In the near future, the interrelations of string theory with both cosmology and elementary particle physics will be put to the test.

In view of this background of both new theoretical ideas and new observations, this book aims at providing a snapshot of current ideas and approaches in string cosmology. The emphasis is put both on presenting theoretical ideas as well as on deriving testable predictions from them. At the same time, the book follows a pedagogical aspect of providing an introduction to present-day research topics for graduate students and scientists of neighboring fields.

The book begins with an introduction to both cosmology and string theory and provides a summary and glossary for the remaining chapters. Subsequently, distinguished string cosmologists present their areas of research. In Chapters 2 and 3, Marco Zagermann and Cliff P. Burgess introduce the important concept of string inflation, emphasizing open string and closed string aspects, respectively. In Chapter 4, Robert C. Myers and Mark Wyman discuss large-scale cosmic superstrings. In Chapter 5, Gary Shiu presents the non-Gaussianities in the power spectrum of cosmic microwave background fluctuations which arise in certain string inflation models. In Chapter 6, Robert Brandenberger introduces string gas cosmology which addresses the question of describing the earliest moments of cosmology, before the standard effective field theory approaches become valid. String gas cosmology also provides an ansatz alternative to inflation. In Chapter 7, Sumit R. Das introduces new approaches to describing spacetime singularities, which are based on gauge–gravity dualities and on matrix models. In Chapter 8, Axel Krause presents the cosmological implications of heterotic M-theory, in particular in view of the dark energy problem and of the generation of gravitational waves.

Though the areas presented are diverse, the book aims at emphasizing the cross-relations between the individual topics, and there are numerous cross-references between the different chapters. As an example consider the power spectrum of fluctuations in the cosmic microwave background: This is defined and introduced in Chapter 1. In Chapters 2 and 3, the form this spectrum assumes in string models of inflation is discussed. In Chapter 5, possible non-Gaussian contributions to this spectrum arising in string inflation models are presented. In Chapter 6, this spectrum is discussed within string gas cosmology, where it assumes a form which differs from the inflationary models in some respects.

It remains to be said that although the book provides an overview over major topics in present-day string cosmology, there are further important concepts in cosmology which, due to the diversity and wealth of the research area, it is not possible to cover here. Nevertheless two of them should be named in this preface: The first is the landscape approach to string vacua. The second is the ekpyrotic scenario of colliding branes, which might provide an alternative to inflation. However, after studying the present book the reader should be equipped with the necessary background information for quickly becoming acquainted with those subjects, too.

Finally, I would like to thank all authors of the individual chapters for their contribution, Martin Ammon for his help in compiling the manuscript, René Meyer for proof-reading, Felix Rust for help with figures, and Anja Tschörtner at Wiley-VCH for her professional handling of the publishing process.

München, September 2008 *Johanna Erdmenger*

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1
Introduction to Cosmology and String Theory

Johanna Erdmenger and Martin Ammon

1.1
Introduction

Cosmology and string theory are two areas of fundamental physics which have progressed significantly over the last 25 years. Joining both areas together provides the possibility of finding microscopic explanations for the history of the early universe on the one hand, and of deriving observational tests for string theory on the other. In the subsequent seven chapters, different aspects of string cosmology are introduced and discussed.

This chapter contains a summary of the basics of both cosmology and string theory in view of providing a reference and glossary for the subsequent chapters. The basic concepts are introduced and briefly described, emphasizing those aspects which are used in the remainder of this book.

There is a wealth of excellent textbooks of both cosmology and string theory, to which readers interested in further details are referred to – for example [1–7]. Reviews on string cosmology include [8–11]. An introduction to string cosmology is found in the textbook [12].

Cosmology is introduced in Sections (1.2)–(1.4) below, and string theory in Sections (1.5)–(1.11).

1.2
Foundations of Cosmology

On the basis of experimental evidence, the common scenario of present-day cosmology is the model of the hot big bang, according to which the universe originated in a hot and dense initial state 13.7 billion years ago, and then has expanded and is still expanding. The most essential feature of the present-day universe is that it is homogeneous and isotropic, that is its structure is the same at every point and in every direction.

This "standard model of cosmology" has received substantial experimental back-up, beginning with the discovery of the cosmic microwave background (CMB) radiation by Penzias and Wilson in 1964 [13]. In recent years, a wealth of precise data has been collected. We list just a few of the important new observations here: In the 1990s, observations of galaxies with the Hubble Space Telescope led in particular to an accurate measurement of the Hubble parameter. Fluctuations in the cosmic microwave background radiations have been observed with the COBE satellite, and subsequently with the BOOMERanG experiment. A further increase in precision came with the WMAP satellite launched in 2001, whose measurements of the parameters of the standard model of cosmology are consistent with the conclusion that the present-day universe is flat. Moreover, these measurements support the scenario of cosmic inflation. They will be supplemented by further data from the PLANCK satellite in the near future.

1.2.1
Metric and Einstein Equations

The homogeneity and isotropy of the universe is best described by the Robertson–Walker metric, which in (–, +, +, +) signature commonly used in string theory reads

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] . \tag{1.1}$$

Here $a(t)$ describes the relative size of space-like hypersurfaces at different times. $\kappa = +1, 0, -1$ stands for positively curved, flat, and negatively curved hypersurfaces, respectively. The frequency of a photon traveling through the expanding universe experiences a redshift z of the size

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a_{\text{present}}}{a_{\text{emitted}}} , \tag{1.2}$$

where λ denotes the photon wavelength.

Using the scale factor $a(t)$ we define the Hubble parameter

$$H \equiv \frac{\dot{a}}{a} , \tag{1.3}$$

with $\dot{a}(t) = da/dt$. As was first discovered and suggested by Edwin Hubble, and has been verified with high precision by modern observational methods, the most distant galaxies recede from us with a velocity given by the Hubble law,

$$v \simeq Hd , \tag{1.4}$$

where d is the distance between us and the galaxies considered.

For describing the expanding universe it is often useful to use the term *e-foldings*, defined as $e \equiv \ln(a(t_f)/a(t_i))$, which describes the growth of the scale factor between some time t_i and a later time t_f.

The dynamics governing the evolution of the scale factor $a(t)$ are obtained from inserting the Robertson–Walker metric into the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} , \tag{1.5}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar, G the Newton constant, and $T_{\mu\nu}$ the energy–momentum tensor. The universe is best described by the perfect fluid form for the energy–momentum tensor of cosmological matter, given by

$$T_{\mu\nu} = (\varrho + p)u_\mu u_\nu + p g_{\mu\nu} , \tag{1.6}$$

where u_μ is the fluid four-velocity, ϱ is the energy density in the rest frame of the fluid, and p is the pressure in the same frame. For consistency with the Robertson–Walker metric, fluid elements are comoving in the cosmological rest frame, with normalized four-velocity

$$u^\mu = (1, 0, 0, 0) . \tag{1.7}$$

The energy–momentum tensor is diagonal and takes the form

$$T_{\mu\nu} = \begin{pmatrix} \varrho & \\ & p g_{ij} \end{pmatrix} , \tag{1.8}$$

where g_{ij} stands for the spatial part of the Robertson–Walker metric, including the factor of $a^2(t)$. Inserting the Robertson–Walker metric (1.1) into the Einstein equation (1.5) with the energy–momentum tensor (1.6), we obtain the first Friedmann equation

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\sum_j \varrho_j - \frac{\kappa}{a^2} , \tag{1.9}$$

with the total energy density $\varrho = \sum_j \varrho_j$, where the sum is over all different types of energy density in the universe. Moreover, we have the evolution equation

$$\frac{\ddot a}{a} + \frac{1}{2}\left(\frac{\dot a}{a}\right)^2 = -4\pi G \sum_j p_j - \frac{\kappa}{2a^2} . \tag{1.10}$$

Here p_j labels the different types of momenta. Equations (1.9) and (1.10) may be combined into the second Friedmann equation

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\sum_j \left(\varrho_j + 3p_j\right) . \tag{1.11}$$

The first Friedmann equation may be used to define the critical energy density

$$\varrho_\mathrm{c} \equiv \left(\sum_j \varrho_j\right)_\mathrm{c} = \frac{3H^2}{8\pi G} \simeq 10^{-29}\,\frac{\mathrm{g}}{\mathrm{cm}^3} , \tag{1.12}$$

for which $\kappa = 0$ and space is flat. The density ratio

$$\Omega_\mathrm{total} \equiv \frac{\varrho}{\varrho_\mathrm{c}} \tag{1.13}$$

thus allows us to relate the total energy density of the universe to its curvature behavior,

$$\begin{aligned} \Omega_{\text{total}} &> 1 \Leftrightarrow \kappa = 1 \,, \\ \Omega_{\text{total}} &= 1 \Leftrightarrow \kappa = 0 \,, \\ \Omega_{\text{total}} &< 1 \Leftrightarrow \kappa = -1 \,. \end{aligned} \tag{1.14}$$

Recent WMAP observations have shown that today, $\Omega_{\text{total}} = 1$ to great accuracy, which leads to the conclusion that the universe is flat.

Energy conservation, $\nabla^\mu T_{\mu\nu} = 0$, gives the relation

$$\dot{\varrho} + 3H(\varrho + p) = 0 \,. \tag{1.15}$$

This relation is not independent of the Friedmann equations. Using both of them, energy conservation (1.15) may be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\varrho a^3\right) = -p\frac{\mathrm{d}}{\mathrm{d}t}a^3 \,. \tag{1.16}$$

1.2.2 Energy Content of the Universe

There is good experimental evidence, in particular from WMAP measurements, that the cosmic fluid contains four different components, and that the total energy density ϱ_{total} in the universe is equal to the critical density ϱ_{c} given by (1.12). This implies

$$\Omega_{\text{total}} = \sum_j \Omega_j = 1 \,, \quad \Omega_j = \frac{\varrho_j}{\varrho_{\text{c}}} \,, \tag{1.17}$$

with Ω_j denoting the present-day fraction of the energy density contributed by the j-th fluid component. The four components of the cosmic fluid are the following:

1. *Radiation*: this component contains predominantly photons, most of which correspond to the cosmic microwave background. The photons are thermally distributed with temperature $T = 2.715$ K. The gas of photons satisfies the equation of state

$$p_{\text{Rad}} = \frac{1}{3}\varrho_{\text{Rad}} \,. \tag{1.18}$$

Moreover, there are also cosmic relic neutrinos in this fluid component, thermally distributed with $T = 1.9$ K. The total energy density of radiation is a small fraction,

$$\Omega_{\text{Rad}} \approx 8 \times 10^{-5} \,, \tag{1.19}$$

of the total present-day energy density.

2. *Baryons*: since their rest mass is much larger than their kinetic energy, their equation of state is

$$p_{\text{B}} \simeq 0 \,. \tag{1.20}$$

Their energy fraction is

$$\Omega_{\mathrm{B}} \approx 4\% \, . \tag{1.21}$$

3. *Dark Matter*: observations of galaxy movement and of matter influence on fluctuations in the CMB provide evidence that there has to be a large amount of long-lived nonrelativistic matter subject to gravitation, which is not detectable by its emitted radiation. Determining the exact structure of this *dark matter* remains one of the essential challenges of modern cosmology. Just as for the baryons, dark matter has the equation of state

$$p_{\mathrm{DM}} \simeq 0 \, , \tag{1.22}$$

while its energy fraction is

$$\Omega_{\mathrm{DM}} \approx 26\% \, , \tag{1.23}$$

so that the overall density of nonrelativistic matter is

$$\Omega_{\mathrm{M}} \equiv \Omega_{\mathrm{B}} + \Omega_{\mathrm{DM}} \approx 30\% \, . \tag{1.24}$$

4. *Dark Energy*: a fourth, similarly unexplained contribution to the cosmic fluid is *dark energy*, which for a total energy density $\Omega = 1$, has to be present in the universe with the large fraction

$$\Omega_{\mathrm{DE}} \approx 70\% \, . \tag{1.25}$$

Its equation of state is expected to be

$$p_{\mathrm{DE}} = -\varrho_{\mathrm{DE}} \, . \tag{1.26}$$

Observational evidence that such a fluid component with negative pressure must be present include tests of the Hubble expansion rate using supernovae which imply that the overall expansion rate of the universe, the Hubble parameter $H = \dot{a}/a$, is increasing at present. The Friedmann equation (1.11) implies that this can only happen for positive energy density if the total pressure is sufficiently negative, $p < -1/3\varrho$. Since none of the other fluid components has negative pressure, a large fraction of such a component must be present.

Each of the above equations of state implies that $w_j = p_j/\varrho_j$ is time independent, with

$$w_{\mathrm{Rad}} = \frac{1}{3} \, , \quad w_{\mathrm{M}} = 0 \, , \quad w_{\mathrm{DE}} = -1 \, . \tag{1.27}$$

Inserting these values into the energy conservation condition in the form (1.16) we obtain, with a_0 the present-day value of a,

$$\varrho_j = \varrho_{j,0} \left(\frac{a_0}{a} \right)^{\alpha_j} \, , \quad \alpha_j = 3(1 + w_j) \, , \tag{1.28}$$

where

$$\alpha_{\text{Rad}} = 4\,, \quad \alpha_{\text{M}} = 3\,, \quad \alpha_{\text{DE}} = 0\,. \tag{1.29}$$

The different equations of state for the different fluid components thus imply that their relative abundances differ in the past universe as compared to the present-day observations since their energy densities vary differently as the universe expands. The history of the universe splits into periods where radiation, matter, and dark energy dominate the evolution of the total density, consecutively. The transition between the radiation and matter-dominated regimes is called *radiation–matter equality* and occurs at a scale given by the comoving wave vector of magnitude $k \simeq (aH)_{\text{eq}}$. Note also that the Friedmann equation (1.9) implies that for $w > -1/3$, the scale factor $a(t)$ grows more slowly than the Hubble scale $H^{-1}(t)$.

It is useful to define the *comoving frame* which moves along with the Hubble flow. A comoving observer is the only one which sees an isotropic universe.

1.2.3 Development of the Universe

During its expansion the universe experienced a number of decisive physical events. The earliest cosmological event for which there is observational evidence is *nucleosynthesis*, which began about three minutes after the big bang, and lasted for about fifteen minutes. At this time, the universe cooled below 1 MeV and light nuclei, hydrogen, helium, lithium, and beryllium, began to accumulate from protons and neutrons. The observational evidence for nucleosynthesis comes from measuring the relative abundance of these elements.

The *radiation–matter crossover* described above occurred at a redshift (1.2) of $z \sim 3600$, or about 50 000 years after the big bang. After this crossover, density inhomogeneities can grow only logarithmically with a while they grow linearly with a during radiation domination.

At a redshift of around $z \sim 1100$, or about 380 000 years after the big bang, *recombination* of nuclei and electrons into electrically neutral atoms occurs. This is the origin of the *cosmic microwave background* which corresponds to the light which is free to move through the universe after recombination. Beforehand, photons interact with the charged medium surrounding them on short scales. The CMB corresponds to a *surface of last scattering* for the photons. Measurements of the CMB temperature fluctuations, which are of the order $\delta T/T \sim 10^{-5}$, provide direct information about the size of primordial density fluctuations at this time.

Finally, *galaxy formation* occurs in the universe once the primordial density fluctuations have been amplified to a scale at which they are no longer well-described by linear perturbations. According to the cold dark matter model (for reviews see for instance [14]), the distribution of galaxies observed today also requires the presence of nonrelativistic (cold) dark matter, together with nonlinear fluctuations.

1.3 Inflation

1.3.1 Puzzles Within the Big Bang Model

When considering the initial conditions characterizing matter in the big bang scenario of an expanding universe, we encounter a number of puzzles. Three of them are discussed in the subsequent text. The initial conditions fix the matter distribution in the universe at the Planckian time of $t_\mathrm{p} = 10^{-43}$ s when classical gravity becomes applicable.

Horizon problem. The horizon problem relates to the fact that the universe is so extremely homogeneous. The Friedmann equation (1.9) implies that the universe expands so quickly that thermal equilibration would violate causality. A dimensional analysis for the ratio a_i/a_0 of the initial and the present-day value of the scale factor $a(t)$ shows that our universe was initially larger than a casual patch by a factor of the order $\dot{a}_\mathrm{i}/\dot{a}_0$. If expansion was always decelerated by an attractive gravity force, which implies $\dot{a}_\mathrm{i}/\dot{a}_0 \gg 1$, then the homogeneity scale was always larger than the causality scale. In fact, using the present size of the universe, the Planck time and the temperature of the universe now and at Planck time, one finds that $\dot{a}_\mathrm{i}/\dot{a}_0 \sim 10^{28}$. This would require an extraordinary fine tuning.

Flatness problem. While the horizon problem relates to the initial conditions for the spatial distribution of matter, the flatness problem relates to the initial velocities. These must satisfy the Hubble law (1.4). The ratio of the kinetic to the total energy of matter in the universe is again given by $(\dot{a}_\mathrm{i}/\dot{a}_0)^2$ and if this ratio is very large, a very unnatural fine tuning between the kinetic energy associated to Hubble expansion and the gravitational potential energy is required. This may be seen from the Friedmann equation (1.9) which implies

$$\Omega(t) - 1 = \frac{\kappa}{(Ha)^2} , \tag{1.30}$$

and thus, since the present-day Ω_0 has been observed to be very close to unity,

$$\Omega_\mathrm{i} - 1 = (\Omega_0 - 1)\left(\frac{\dot{a}_0}{\dot{a}_\mathrm{i}}\right)^2 < 10^{-56} \tag{1.31}$$

for $\dot{a}_\mathrm{i}/\dot{a}_0 \sim 10^{28}$. Such an astonishing fine-tuning appears implausible.

Initial perturbations. A third puzzle, related to the other two, concerns the origin of the original inhomogeneities needed to explain the large-scale structure of the present-day universe.

1.3.2 The Concept of Inflation

A concept which can solve the puzzles mentioned is inflation. The idea of inflation, first suggested in [15], is that there is an initial stage of accelerated expansion where gravity acts as a repulsive force. If gravity was always positive, then $\dot{a}_\mathrm{i}/\dot{a}_0$ is necessarily larger than one since gravity decelerates expansion. $\dot{a}_\mathrm{i}/\dot{a}_0 < 1$ is possible only if gravity is repulsive during some period of expansion. This period of repulsive gravity can in particular explain the creation of our universe from a single causally connected region. Moreover, since it accelerates expansion, small initial velocities inside a causally connected region become very large.

In the inflationary period we have $\ddot{a} > 0$. From the Friedmann equation (1.11), which may be written in the form

$$\ddot{a} = -\frac{4\pi G}{3}(\varrho + 3p)a \;, \tag{1.32}$$

for the total energy density ϱ and the total momentum p. We read off that $\ddot{a} > 0$ requires $\varrho + 3p < 0$. This implies that the strong energy dominance condition, $\varrho + 3p > 0$, must be violated during inflation. One example which violates this condition is a positive cosmological constant for which $p \approx -\varrho$.

Inflation can only appear during a limited period in time for consistency with cosmological observations. In simple inflationary models it takes place in the period of $t_\mathrm{inf} \sim 10^{-36} - 10^{-34}$ s after the big bang. It must end with a "graceful exit", after which $\ddot{a}$ becomes negative again.

Let us consider how the condition $p = -\varrho$ may be realized. The matter and its interactions during inflation is simply modeled by a single scalar field $\varphi = \varphi(t)$, the inflaton. This can be viewed as an order parameter describing the vacuum of the physical theory determining the very high energy physics. This gives rise to

$$\varrho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \;, \qquad p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \;, \tag{1.33}$$

with potential $V(\varphi)$. The condition $p \approx -\varrho$ requires $\dot{\varphi}^2 \ll V(\varphi)$, so the kinetic energy must be smaller than the potential energy. This is referred to as "slow roll". The Klein–Gordon equation or $\dot{\varrho} = -3H(\varrho + p)$ imply

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0 \;, \quad V' \equiv \frac{\partial V}{\partial \varphi} \;. \tag{1.34}$$

The second term in this equation corresponds to a friction term proportional to H. Generically if friction becomes large, we may neglect the second derivative term $\ddot{\varphi}$ and find an approximate asymptotic solution to (1.34). With $\ddot{\varphi} = 0$, (1.34) implies

$$\dot{\varphi} \approx -\left(\frac{V'}{3H}\right) . \tag{1.35}$$

From the slow-roll condition $1/2\dot{\varphi}^2 \ll V$ we then obtain the two conditions

$$\varepsilon \ll 1, \quad \eta \ll 1 \tag{1.36}$$

for the slow-roll parameters

$$\varepsilon \equiv \frac{1}{2}\left(\frac{M_\mathrm{p} V'}{V}\right)^2, \quad \eta \equiv \frac{M_\mathrm{p}^2 V''}{V} \;. \tag{1.37}$$

Single-field slow-roll inflation leads to important consequences for density fluctuations, as discussed in Section 1.4.3 below, and in further detail in Chapters 2 and 3. It turns out that these consequences can be described just in terms of the two small parameters ε and η, together with the value of the Hubble parameter during inflation.

1.4 Fluctuations

1.4.1 Characterization of Small Fluctuations

An important question of cosmology is to study how the large-scale structure of the universe which is observed today, including galaxies and clusters of galaxies, developed from the initially flat and homogeneous universe. The large-scale structure has evolved from initially small fluctuations during the expansion of the universe. These small fluctuations are taken as initial conditions of the big bang model.

In linear approximation, the fluctuations of the Robertson–Walker metric (1.1), given by

$$\mathrm{d}s^2 = \left[{}^{(0)}g_{\mu\nu} + \delta g_{\mu\nu}(x)\right] \mathrm{d}x^\mu \,\mathrm{d}x^\nu \ , \tag{1.38}$$

may be decomposed as follows using the symmetry properties of the unperturbed Robertson–Walker metric. The linear approximation applies to fluctuations on length scales below the Hubble scale. It implies that the different fluctuation modes decouple and have a Gaussian distribution.

The δg_{00} component has the form

$$\delta g_{00} = 2a^2\phi \ , \tag{1.39}$$

with scalar ϕ. The spacetime component δg_{0i} has the form

$$\delta g_{0i} = a^2(\partial_i B + S_i) \ , \tag{1.40}$$

where the index i runs over the three space-like components. The vector S_i satisfies $\partial_i S^i = 0$. The fluctuation component δg_{ij} is a tensor under 3-rotations and may be written as

$$\delta g_{ij} = a^2(2\psi\delta_{ij} + 2\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}) \ , \tag{1.41}$$

where ψ, E are scalars, $\partial_i F^i = 0$, $h^i_i = 0$, $\partial_i h^i_j = 0$.

The fluctuations are thus described by the *scalar fluctuations* ϕ, ψ, B, E, the *vector fluctuations* S_i, F_i, and the *tensor fluctuations* h_{ij}. The latter described gravitational waves. All of these functions change under coordinate reparametrizations, but may be regrouped into coordinate invariant expressions. A particularly important coordinate invariant combination of scalar fluctuations is

$$\Phi \equiv \phi - \frac{1}{a}\left[a\,(B - E')\right]' \ , \tag{1.42}$$

where the prime denotes differentiation with respect to conformal time η, defined by $\mathrm{d}t = a(t)\,\mathrm{d}\eta$. The fluctuation Φ is the relativistic equivalent of the Newton potential. The Einstein equation (1.5) relates the metric fluctuations to the energy–momentum tensor and its fluctuations.

1.4.2
Power Spectrum

The power spectrum of fluctuations $P(k)$ is obtained by transforming to Fourier space. In the linear approximation, a nonrelativistic Fourier transformation is appropriate (see for instance [16]). In particular, for the scalar fluctuations Φ of (1.42) we have

$$\Phi(\boldsymbol{r}, t) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \Phi_k(t) \exp\left[\mathrm{i}(\boldsymbol{k}/a) \cdot \boldsymbol{r}\right] , \tag{1.43}$$

where homogeneity and isotropy of the background imply that $\Phi_k(t)$ depends only on $k = |\boldsymbol{k}|$ and t. $\boldsymbol{k}$ is the wave vector in the comoving frame which moves along with the Hubble flow. The physical wavelength is $\lambda = 2\pi a/k$. The power spectrum is obtained from the *autocorrelation function* $\xi_\Phi(\boldsymbol{r})$,

$$\xi_\Phi(\boldsymbol{r}) \equiv \langle \Phi(\boldsymbol{r})\Phi(0) \rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} P_\mathrm{s}(k) \exp\left[\mathrm{i}(\boldsymbol{k}/a) \cdot \boldsymbol{r}\right] . \tag{1.44}$$

If we assume that the average denoted by the brackets $\langle\rangle$ is given by a Gaussian distribution, we have

$$P_\mathrm{s}(k) \equiv |\Phi(k)|^2 \tag{1.45}$$

for the power spectrum. A dimensionless measure of the power spectrum is obtained by performing an angular integration within the Fourier transformation,

$$\langle \Phi(\boldsymbol{r})\Phi(0) \rangle = \int_0^\infty \frac{\mathrm{d}k}{k} \Delta_\Phi^2(k) \frac{\sin(kr/a)}{kr/a} , \tag{1.46}$$

with

$$\Delta_\mathrm{s}^2(k) \equiv \frac{1}{2\pi^2} k^3 P_\mathrm{s}(k) . \tag{1.47}$$

The *spectral index* n_s is defined by

$$n_\mathrm{s} - 1 \equiv \frac{\mathrm{d} \ln \Delta_\mathrm{s}^2}{\mathrm{d} \ln k} . \tag{1.48}$$

A spectral index of $n_s = 1$ corresponds to a scale invariant spectrum, also called a *Harrison–Zel'dovich* spectrum. For tensor fluctuations or gravitational waves, denoted by h_{ij} in (1.41), the spectral index is defined by

$$n_T \equiv \frac{d \ln \Delta_T^2}{d \ln k} \,. \tag{1.49}$$

These fluctuations have not yet been observed.

The structure of the spectrum is influenced by the expansion of the universe. For instance, for a spectrum which is scale invariant for modes $k < k_{eq}$, with $k_{eq} \equiv (aH)_{eq}$ the momentum scale at radiation–matter equality, we have a spectrum of the form $\Delta_\Phi^2(k) \propto 1/k^4$ for modes $k > k_{eq}$. This behavior arises since modes with $k > k_{eq}$ re-enter the Hubble scale before radiation–matter equivalence, while modes with $k < k_{eq}$ do so afterwards (remember that aH shrinks during both matter and radiation dominated periods).

1.4.3 Fluctuations and Inflation

Inflation has a significant impact on the fluctuation spectrum which originates from two facts. First, there are new contributions to the equations of motion for the metric fluctuations which originate from fluctuations of the scalar inflaton field. Second, while the scale aH shrinks during matter and radiation dominance, it grows during inflation, such that length scales $L \sim a$ grow faster than H^{-1}. There is a horizon corresponding to the graceful exit at which inflation ends, and where the length scales grow slower than the Hubble scale again.

In inflationary models, the fluctuations χ of the inflaton ϕ are obtained from linearizing the equation of motion

$$\Box \phi - V'(\phi) = 0 \tag{1.50}$$

after replacing $\phi \to \phi + \chi$ and linearizing in χ as well as in the metric fluctuations. The time dependence of the background forces χ and the scalar metric fluctuation Φ to mix with each other. For instance, for $k \gg aH$ the solution for Φ of the coupled equations shows a damped oscillation. In the opposite regime $k \ll aH$, the coupled fluctuations equations read, in the slow-roll approximation,

$$3H\dot{\chi} + V''(\phi)\chi + 2V'(\phi)\Phi = 0 \,, \qquad 2M_P^2 H\Phi = \dot{\phi}\chi \,. \tag{1.51}$$

The solutions are, after transforming to momentum space,

$$\chi_k = C_k \frac{V'(\phi)}{V(\phi)} \,, \quad \Phi_k = -\frac{C_k}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \,, \tag{1.52}$$

with C_k a constant of integration. This constant is set by the initial conditions at the horizon where the universe exits the inflation period. Since all classical fluctuations are damped away during the inflation period, the new perturbations are fueled by quantum fluctuations. The quantization for the inflaton perturbations reads

$$\chi(x) = \int \frac{d^3 \boldsymbol{k}}{(2\pi)^3} \left[c_k u_k(t) \exp(i\boldsymbol{k} \cdot \boldsymbol{r}/a) + c_k^* u_k^*(t) \exp(-i\boldsymbol{k} \cdot \boldsymbol{r}/a) \right] , \tag{1.53}$$

where c_k^*, c_k are the creation and annihilation operators and $u_k(t) \exp(i\boldsymbol{k}\cdot\boldsymbol{r}/a)$ is a basis of eigenmodes of the background field equation. Evaluating χ_k at the horizon exit t_{he}, where $k = aH$, determines the integration constant to be

$$C_k = u_k(t_{\mathrm{he}}) \left(\frac{V}{V'} \right)_{\mathrm{he}} . \tag{1.54}$$

Using this, the result for the power spectrum for the scalar metric fluctuations eventually reads

$$\Delta_\Phi^2(k) = \left(\frac{V}{24\pi^2 M_{\mathrm{P}}^4 \varepsilon} \right)_{\mathrm{he}} , \tag{1.55}$$

with ε the slow-roll parameter defined in (1.37).

For the spectral index (1.48) we obtain, using that

$$\frac{d}{d\ln k} = -M_{\mathrm{P}}^2 \left(\frac{V'}{V} \right) \frac{d}{d\varphi} \tag{1.56}$$

in the slow-roll approximation,

$$n_{\mathrm{s}} - 1 = -6\varepsilon + 2\eta , \tag{1.57}$$

with ε, η as in (1.37), where the right hand side is evaluated at horizon exit. This implies that $n_{\mathrm{s}} < 1$. Therefore, inflation predicts a *red tilt* in the scalar power spectrum, since $n_{\mathrm{s}} < 1$ means that the amplitude for smaller momentum modes is larger than the amplitude for larger momentum modes. Similarly, for tensor modes inflation predicts that

$$\Delta_{\mathrm{T}}^2(k) = \frac{2V}{3\pi^2 M_{\mathrm{P}}^4} , \tag{1.58}$$

and

$$n_{\mathrm{T}} = -2\varepsilon \tag{1.59}$$

for the tensor spectral index (1.49).

This concludes our brief introduction to cosmology.

1.5 Bosonic String Theory

We now turn to the essentials of string theory, emphasizing aspects relevant to the subsequent chapters. We begin by discussing bosonic string theory, after which we focus on superstring theories and dualities between these theories. Moreover, the unification of consistent superstring theories in ten dimensions into M-theory will be discussed. Also, there is a short introduction to D-branes and to compactification scenarios of string theory to four spacetime dimensions. Finally, short introductions to string thermodynamics and to the AdS/CFT correspondence are given.

Whereas in conventional quantum field theory the elementary particles are point-like objects, the fundamental objects in perturbative string theory are one-dimensional strings. As a string evolves in time, it sweeps out a two-dimensional surface in spacetime, the *world-sheet* of the string, which is the string counterpart of the world-line for a point particle. To parametrize the world-sheet of the string, two parameters are needed: the world-sheet time coordinate $\tau = \sigma^0$, which parametrizes the world-line in the case of a point-like particle, and $\sigma = \sigma^1$ parametrizing the spatial extent of the string. The embedding of the world-sheet of the fundamental string into the *(target) spacetime* is given by the functions $X^\mu(\tau, \sigma)$, which are also referred to as the embedding functions or target spacetime string coordinates. Since the action of a point-like particle is given by the length of the world-line, the natural generalization to the action of a string propagating through flat spacetime is given by the area of the world-sheet,

$$S = -\frac{1}{2\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{-\det \partial_\alpha X^\mu \partial_\beta X_\mu} \,, \tag{1.60}$$

where $\mathrm{d}^2\sigma = \mathrm{d}\sigma^0 \mathrm{d}\sigma^1 = \mathrm{d}\tau\,\mathrm{d}\sigma$. This is the *Nambu–Goto action* of a fundamental string. The determinant is taken with respect to $\alpha, \beta = 0, 1$, where α and β label the world-sheet coordinates. Moreover, we use the short-hand notation $\partial_\alpha = \partial/\partial\sigma^\alpha$. The only free parameter appearing in this action is α', which is related to the length of the string, $\alpha' = l_s^2$. The dimensionful prefactor $T = 1/(2\pi\alpha')$ can be interpreted as the string tension or the energy per length. To get rid of the square root in the action of the fundamental string in view of quantization, an auxiliary field $h_{\alpha\beta}(\sigma^0, \sigma^1)$ is introduced, which has to satisfy the constraints given below. This gives rise to the *Polyakov action*,

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \,, \tag{1.61}$$

which is classically equivalent to (1.60) using the equations of motion of $h_{\alpha\beta}$. In (1.61), h is the determinant of the matrix $h_{\alpha\beta}$ and $h^{\alpha\beta}$ is the inverse matrix of $h_{\alpha\beta}$, that is $h^{\alpha\beta} h_{\beta\gamma} = \delta^\alpha_\gamma$. The auxiliary field $h_{\alpha\beta}$ is called the *world-sheet metric*. The Polyakov action is invariant under the following symmetries:

– *Poincaré transformations*
  These transformations are global symmetries of the world-sheet fields X^μ of the form

$$\delta X^\mu = \Lambda^\mu{}_\nu X^\nu + a^\mu \qquad \text{and} \qquad \delta h^{\alpha\beta} = 0 \,, \tag{1.62}$$

  where Λ^μ_ν and a^μ are Lorentz transformations and spacetime translations, respectively.

- *Reparametrizations*
  The Polyakov action is invariant under reparametrizations since a change in the world-sheet parametrization of the form $\sigma^\alpha \to f^\alpha(\sigma) = \sigma'^\alpha$ with

$$h_{\alpha\beta}(\sigma) = \frac{\partial f^\gamma}{\partial \sigma^\alpha}\frac{\partial f^\delta}{\partial \sigma^\beta} h_{\gamma\delta}(\sigma') \qquad \text{and} \qquad X'^\mu(\tau', \sigma') = X^\mu(\tau, \sigma) \tag{1.63}$$

  does not change the action.
- *Weyl transformations*
  The action is also invariant under rescalings of the world-sheet metric $h_{\alpha\beta}$

$$h_{\alpha\beta} \to \mathrm{e}^{\omega(\sigma,\tau)} h_{\alpha\beta} \qquad \text{and} \qquad \delta X^\mu = 0\,. \tag{1.64}$$

  Since this transformation is a local symmetry of the action, the energy–momentum tensor of the field theory defined on the world-sheet is traceless, that is $T^a_a = 0$. After quantization, Weyl Symmetry is potentially broken by a conformal anomaly. In string theory, this anomaly has to be absent, which is only the case if the spacetime dimension of the target space is $D = 26$ for bosonic string theory. Moreover, there are restrictions on the form of the background fields allowed (see Section 1.5.4).

The local symmetries may be used to choose a gauge which brings the components of the world-sheet metric into a simple form. In particular, the equations of motion of the action can be simplified by choosing the gauge

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{1.65}$$

In this and other conformal gauges, the equation of motion for $X^\mu(\tau, \sigma)$ is a relativistic wave equation,

$$\left(\partial_\tau^2 - \partial_\sigma^2\right) X^\mu = 0\,, \tag{1.66}$$

supplemented by the *Virasoro constraints*

$$\partial_\tau X^\mu \partial_\sigma X_\mu = 0\,, \tag{1.67}$$

$$\partial_\tau X^\mu \partial_\tau X_\mu - \partial_\sigma X^\mu \partial_\sigma X_\mu = 0\,. \tag{1.68}$$

These constraints are derived from the equations of motion of the auxiliary field $h_{\alpha\beta}$ in the Polyakov action and have to be satisfied to ensure the equivalence of the two actions (1.60) and (1.61) at the classical level.

1.5.1
Open and Closed Strings

By applying variational methods, it is possible to derive not only the equations of motion but also the possible boundary conditions for the string. There are two different types of strings: open and closed strings.

1.5.1.1
Closed Strings

Closed strings are topologically equivalent to a circle, that is they do not have endpoints. If we parametrize these strings by the parameter $\sigma \in [0, 2\pi[$, the boundary conditions read

$$X^\mu(\tau, 0) = X^\mu(\tau, 2\pi), \quad \partial_\sigma X^\mu(\tau, 0) = \partial_\sigma X^\mu(\tau, 2\pi), \quad h^{\alpha\beta}(\tau, 0) = h^{\alpha\beta}(\tau, 2\pi)\ . \tag{1.69}$$

This means that the string coordinates X^μ are periodic, that is the endpoints are joined to form a closed loop. The mode expansion for the closed string is simply given by a pair of left and right-moving waves, which travel around the string in opposite directions,

$$X^\mu(\tau, \sigma) = X^\mu_{\mathrm{R}}(\tau - \sigma) + X^\mu_{\mathrm{L}}(\tau + \sigma)\ . \tag{1.70}$$

X_{R} (X_{L}) are the *right (left) moving parts*, respectively. The mode decompositions of the left and right-moving parts are given by

$$X^\mu_{\mathrm{R}}(\tau - \sigma) = \frac{1}{2}x^\mu_0 + \alpha' p^\mu_{\mathrm{R}}(\tau - \sigma) + \mathrm{i}\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n}\alpha^\mu_n\, \mathrm{e}^{-2\mathrm{i}n(\tau-\sigma)} \tag{1.71}$$

and

$$X^\mu_{\mathrm{L}}(\tau + \sigma) = \frac{1}{2}x^\mu_0 + \alpha' p^\mu_{\mathrm{L}}(\tau + \sigma) + \mathrm{i}\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n}\tilde{\alpha}^\mu_n\, \mathrm{e}^{-2\mathrm{i}n(\tau+\sigma)}\ . \tag{1.72}$$

x^μ_0 and p^μ are the center-of-mass position and momentum of the string, respectively. The periodicity condition requires that $p^\mu_{\mathrm{R}} = p^\mu_{\mathrm{L}}$, and reality of X^μ requires the conditions $\alpha^\mu_{-n} = (\alpha^\mu_n)^\star$ and $\tilde{\alpha}^\mu_{-n} = (\tilde{\alpha}^\mu_n)^\star$. Moreover, the center-of-mass momentum p^μ can be identified with the zero mode of the expansion by

$$\alpha^\mu_0 = \tilde{\alpha}^\mu_0 = \sqrt{\frac{\alpha'}{2}}p^\mu\ . \tag{1.73}$$

1.5.1.2
Open Strings

For open strings, two different boundary conditions in each direction μ of the spacetime are possible, Neumann or Dirichlet boundary conditions. In the case of *Neumann* boundary conditions, the component of the momentum normal to the boundary of the world-sheet vanishes, that is

$$\partial_\sigma X_\mu(\tau, 0) = \partial_\sigma X_\mu(\tau, \pi) = 0\ . \tag{1.74}$$

Note that the open string is now parametrized by $\sigma \in [0, \pi]$. The boundary condition implies that there is no momentum flowing through the ends of the string. The mode decomposition of the embedding function $X^\mu(\tau, \sigma)$ is given by

$$X^\mu(\tau, \sigma) = x^\mu_0 + 2\alpha' p^\mu \tau + \mathrm{i}\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n}\alpha^\mu_n\, \mathrm{e}^{-\mathrm{i}n\tau} \cos(n\sigma)\ . \tag{1.75}$$