INTRODUCTION TO RADIOLOGICAL PHYSICS AND RADIATION DOSIMETRY

FRANK HERBERT ATTIX
Professor of Medical Physics
University of Wisconsin Medical School
Madison, Wisconsin

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This book is dedicated to my parents
Ulysses Sheldon Attix and Alma Katherine Attix (nee Michelsen),
my wife Shirley Adeline Attix (nee Lohr),
my children Shelley Anne and Richard Haven,
and to radiological physics students everywhere

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Preface

This book is intended as a text for an introductory course at the graduate or senior undergraduate level. At the University of Wisconsin this is a three-credit course: Medical Physics 501—Radiological Physics and Dosimetry, consisting of about 45 lectures and 15 problem discussion sessions, each 50 minutes in length. By moving along briskly and by scheduling the exams at other times, the material in the book can be adequately covered in one semester. The chapters are designed to be taught in sequence from 1 through 16.

The book is written on the assumption that the student has previously studied integral calculus and atomic or modern physics. Thus integrals are used without apology wherever necessary, and no introductory chapter to review atomic structure and elementary particles is provided. Chapter 1 in Johns and Cunningham's book *The Physics of Radiology*, 3rd or 4th edition, for example, can be used for remedial review if needed.

The present text is pragmatic and classical in approach, not necessarily developing equations from first principles, as is more often done by Anderson (1984) in his admirable book *Absorption of Ionizing Radiation*. Missing details and derivations that are relevant to interaction processes may be found there, or in the incomparable classic *The Atomic Nucleus* by Robley Evans, recently republished by Krieger.

A challenging problem in writing this book was how to limit its scope so that it would fit a coherent course that could be taught in one semester and would not reach an impractical and unpublishable length. It had to be in a single volume for convenient use as a text, as it was not intended to be a comprehensive reference like the three-volume second edition of *Radiation Dosimetry*, edited by Attix, Roesch, and Tochilin. Although that treatise has been used for textbook purposes in some courses, it was never intended to be other than a reference. In limiting the scope of this text the following topic areas were largely omitted and are taught as separate courses in the University of Wisconsin Department of Medical Physics: radiotherapy physics, nuclear medicine, diagnostic radiological physics, health physics (radiation protection), and radiobiology. Other texts are used for those courses. Radiation-generating equipment is described in the courses on radiotherapy and diagnostic physics, as the design of such equipment is specific to its use.

What is included is a logical, rather than historical, development of radiological physics, leading into radiation dosimetry in its broadest sense. There is no such thing as a *perfect* sequence—one that always builds on material that has gone before and never has to reach ahead for some as yet untaught fact. However, the present order of chapters has evolved from several years of trial-and-error classroom testing and works quite well.

A few specifics deserve mentioning:

Extensive, but not exclusive, use is made of SI units. The older units in some instances offer advantages in convenience, and in any case they are not going to vanish down a "memory hole" into oblivion. The rad, rem, roentgen, curie, and erg will remain in the existing literature forever, and we should all be familiar with them. There is, moreover, no reason to restrain ourselves from using centimeters or grams when nature provides objects for which convenient-sized numbers will result. I believe that units should be working for us, not the other way around.

The recommendations of the International Commission on Radiation Units and Measurements (ICRU) are used as the primary basis for the radiological units in this book, as far as they go. However, additional quantities (e.g., collision kerma, energy transferred, net energy transferred) have been defined where they are needed in the logical development of radiological physics.

Several important concepts have been more clearly defined or expanded upon, such as radiation equilibrium, charged-particle equilibrium, transient charged-particle equilibrium, broadbeam attenuation, the reciprocity theorem (which has been extended to homogeneous but nonisotropic fields), and a rigorous derivation of the Kramers x-ray spectrum.

Relegating neutron dosimetry to the last chapter is probably the most arbitrary and least logical chapter assignment. Initially it was done when the course was taught in two halves, with the first half alone being prerequisite for radiotherapy physics. Time constraints and priorities dictated deferring all neutron considerations until the second half. Now that the course (and text) has been unified, that reason is gone, but the neutron chapter remains number 16 because it seems to fit in best after all the counting detectors have been discussed. Moreover it provides an appropriate setting for introducing microdosimetry, which finds its main application in characterizing neutron and mixed n–γ fields.

The tables in the appendixes have been made as extensive as one should hope to find in an introductory text. The references for all the chapters have been collected together at the back of the book to avoid redundancy, since some references are repeated in several chapters. Titles of papers have been included. A comprehensive table of contents and index should allow the easy location of material.

For the authors-to-be among this book's readers: This book was begun in 1977 and completed in 1986. It started from classroom notes that were handed out to students to supplement other texts. These notes gradually evolved into chapters that were modified repeatedly, to keep what worked with the students, and change what didn't. This kind of project is not for anyone with a short attention span.

The original illustrations for this book were drawn by F. Orlando Canto. Kathryn A. McSherry and Colleen A. Schutz of the office staff were very helpful. I also thank the University of Wisconsin Department of Medical Physics for allowing me to use their copying equipment.

Finally, it is a pleasure to acknowledge that the preparation of this book could not have been accomplished without the dedicated partnership and enthusiasm of my wife Shirley. Not only did she do all the repetitious typing, during a time before a word processor was available, but she never complained about the seemingly endless hours I spent working on it.

HERB ATTIX

Madison, Wisconsin
August 1986

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CHAPTER 1
Ionizing Radiation

I. INTRODUCTION

Radiological physics is the science of ionizing radiation and its interaction with matter, with special interest in the energy thus absorbed. Radiation dosimetry has to do with the quantitative determination of that energy. It would be awkward to try to discuss these matters without providing at the outset some introduction to the necessary concepts and terminology.

Radiological physics began with the discovery of x-rays by Wilhelm Röntgen, of radioactivity by Henri Becquerel, and of radium by the Curies in the 1890s. Within a very short time both x-rays and radium became useful tools in the practice of medicine. In fact, the first x-ray photograph (of Mrs. Röntgen's hand) was made by Röntgen late in 1895, within about a month of his discovery, and physicians on both sides of the Atlantic were routinely using x-rays in diagnostic radiography within a year, thus setting some kind of record for the rapid adoption of a new technology in practical applications.

The historical development of the science of radiological physics since then is itself interesting, and aids one in understanding the quantities and units used in this field today. However, such an approach would be more confusing than helpful in an introductory course. Historical reviews have been provided by Etter (1965), Parker and Roesch (1962), and by Roesch and Attix (1968).

II. TYPES AND SOURCES OF IONIZING RADIATIONS

Ionizing radiations are generally characterized by their ability to excite and ionize atoms of matter with which they interact. Since the energy needed to cause a valence electron to escape an atom is of the order of 4–25 eV, radiations must carry kinetic or quantum energies in excess of this magnitude to be called "ionizing." As will be seen from Eq. (1.1), this criterion would seem to include electromagnetic radiation with wavelengths up to about 320 nm, which includes most of the ultraviolet (UV) radiation band ($\sim$10–400 nm). However, for practical purposes these marginally ionizing UV radiations are not usually considered in the context of radiological physics, since they are even less capable of penetrating through matter than is visible light, while other ionizing radiations are generally more penetrating.

The personnel hazards presented by optical lasers and by radiofrequency (RF) sources of electromagnetic radiation are often administratively included in the area of a health physicist's responsibilities, together with ionizing radiation hazards. Moreover, the determination of the energy deposition in matter by these radiations is often referred to as "dosimetry". However, the physics governing the interaction of such radiations with matter is totally different from that for *ionizing* radiations, and this book will not deal with them.

The important types of ionizing radiations to be considered are:

1. *γ-rays:* Electromagnetic radiation emitted from a nucleus or in annihilation reactions between matter and antimatter. The quantum energy of any electromagnetic photon is given in keV by

$$E_\gamma = h\nu = \frac{hc}{\lambda} = \frac{12.398 \text{ keV-Å}}{\lambda} = \frac{1.2398 \text{ keV-nm}}{\lambda} \tag{1.1}$$

where 1 Å (Angstrom) = 10^{-10} m, Planck's constant is

$$h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-18} \text{ keV s}$$

(note that 1.6022×10^{-16} J = 1 keV), and the velocity of light *in vacuo* is

$$c = 2.998 \times 10^{8} \text{ m/s} = 2.998 \times 10^{18} \text{ Å/s} = 2.998 \times 10^{17} \text{ nm/s}$$

Evidently, by Eq. (1.1) the quantum energy of a photon of 0.1-nm wavelength is 12.4 keV, within one part in 6000.

The practical range of photon energies emitted by radioactive atoms extends from 2.6 keV (K_α characteristic x-rays from electron capture in $^{37}_{18}$Ar) to the 6.1- and 7.1-MeV γ-rays from $^{16}_{7}$N.

2. *X-rays:* Electromagnetic radiation emitted by charged particles (usually electrons) in changing atomic energy levels (called *characteristic* or *fluorescence* x-rays) or in slowing down in a Coulomb force field (*continuous* or *bremsstrahlung* x-rays). Note that an x-ray and a γ-ray photon of a given quantum energy have identical properties, differing only in mode of origin. Older texts sometimes referred to all lower-energy photons as x-rays and higher energy photons as γ-rays, but this basis for the distinction is now obsolete. Most commonly, the energy ranges of x-rays are now referred to as follows, in terms of the generating voltage:

0.1–20 kV	Low-energy or "soft" x-rays, or "Grenz rays"
20–120 kV	Diagnostic-range x-rays
120–300 kV	Orthovoltage x-rays
300 kV–1 MV	Intermediate-energy x-rays
1 MV upward	Megavoltage x-rays

3. *Fast Electrons:* If positive in charge, they are called positrons. If they are emitted from a nucleus they are usually referred to as β-rays (positive or negative). If they result from a charged-particle collision they are referred to as "δ-rays". Intense continuous beams of electrons up to 12 MeV are available from Van de Graaff generators, and pulsed electron beams of much higher energies are available from linear accelerators ("linacs"), betatrons, and microtrons. Descriptions of such accelerators, as encountered in medical applications, have been given by Johns and Cunningham (1974) and Hendee (1970).

4. *Heavy Charged Particles:* Usually obtained from acceleration by a Coulomb force field in a Van de Graaff, cyclotron, or heavy-particle linear accelerator. Alpha particles are also emitted by some radioactive nuclei. Types include:

- Proton—the hydrogen nucleus.
- Deuteron—the deuterium nucleus, consisting of a proton and neutron bound together by nuclear force.
- Triton—a proton and two neutrons similarly bound.
- Alpha particle—the helium nucleus, i.e., two protons and two neutrons. ^{3}He particles have one less neutron.
- Other heavy charged particles consisting of the nuclei of heavier atoms, either fully stripped of electrons or in any case having a different number of electrons than necessary to produce a neutral atom.
- Pions—negative π-mesons produced by interaction of fast electrons or protons with target nuclei.

5. *Neutrons:* Neutral particles obtained from nuclear reactions [e.g., (p, n) or fission], since they cannot themselves be accelerated electrostatically.

The range of kinetic or photon energies most frequently encountered in applications of ionizing radiations extends from 10 keV to 10 MeV, and relevant tabulations of data on their interactions with matter tend to emphasize that energy range. Likewise the bulk of the literature dealing with radiological physics focuses its attention primarily on that limited but useful band of energies. Recently, however, clinical radiotherapy has been extended (to obtain better spatial distribution, and/or more direct cell-killing action with less dependence on oxygen) to electrons and x-rays up to about 50 MeV; and neutrons to 70 MeV, pions to 100 MeV, protons to 200 MeV, α-particles to 10^3 MeV, and even heavier charged particles up to 10 GeV are being investigated in this connection. Electrons and photons down to about 1 keV are also proving to be of experimental interest in the context of radiological physics.

The ICRU (International Commision on Radiation Units and Measurements, 1971) has recommended certain terminology in referring to ionizing radiations which emphasizes the gross differences between the interactions of charged and uncharged radiations with matter:

1. *Directly Ionizing Radiation.* Fast charged particles, which deliver their energy to matter directly, through many small Coulomb-force interactions along the particle's track.
2. *Indirectly Ionizing Radiation.* X- or γ-ray photons or neutrons (i.e., uncharged particles), which first transfer their energy to charged particles in the matter through which they pass in a relatively few large interactions. The resulting fast charged particles then in turn deliver the energy to the matter as above.

It will be seen that the deposition of energy in matter by indirectly ionizing radiation is thus a *two-step process*. In developing the concepts of radiological physics the importance of this fact will become evident.

The reason why so much attention is paid to ionizing radiation, and that an extensive science dealing with these radiations and their interactions with matter has evolved, stems from the unique effects that such interactions have upon the irradiated material. Biological systems (e.g., humans) are particularly susceptible to damage by ionizing radiation, so that the expenditure of a relatively trivial amount of energy (~ 4 J/kg) throughout the body is likely to cause death, even though that amount of energy can only raise the gross temperature by about 0.001°C. Clearly the ability of ionizing radiations to impart their energy to individual atoms, molecules, and biological cells has a profound effect on the outcome. The resulting high local concentrations of absorbed energy can kill a cell either directly or through the formation of highly reactive chemical species such as free radicals* in the water medium that constitutes the bulk of the biological material. Ionizing radiations can also produce gross changes, either desirable or deleterious, in organic compounds by breaking molecular bonds, or in crystalline materials by causing defects in the lattice structure.

*A free radical is an atom or compound in which there is an unpaired electron, such as H or CH_3.

Even structural steel will be damaged by large enough numbers of fast neutrons, suffering embrittlement and possible fracture under mechanical stress.

Discussing the details of such radiation effects lies beyond the scope of this book, however. Here we will concentrate on the basic physics of the interactions, and methods for measuring and describing the energy absorbed in terms that are useful in the various applications of ionizing radiation.

III. DESCRIPTION OF IONIZING RADIATION FIELDS

A. Consequences of the Random Nature of Radiation

Suppose we consider a point P in a field of ionizing radiation, and ask: "How many *rays* (i.e., photons or particles) will strike P per unit time?" The answer is of course zero, since a point has no cross-sectional area with which the rays can collide. Therefore, the first step in describing the field at P is to associate some nonzero volume with the point. The simplest such volume would be a sphere centered at P, as shown in Fig. 1.1, which has the advantage of presenting the same cross-sectional target area to rays incident from all directions. The next question is how large this imaginary sphere should be. That depends on whether the physical quantities we wish to define with respect to the radiation field are *stochastic* or *nonstochastic*.

A stochastic quantity has the following characteristics:*

a. Its values occur randomly and hence cannot be predicted. However, the probability of any particular value is determined by a probability distribution.
b. It is defined for finite (i.e. noninfinitesimal) domains only. Its values vary discontinuously in space and time, and it is meaningless to speak of its gradient or rate of change.
c. In principle, its values can each be measured with an arbitrarily small error.
d. The *expectation value* N_e of a stochastic quantity is the mean $\overline{N}$ of its measured values N as the number n of observations approaches ∞. That is, $\overline{N} \rightarrow N_e$ as $n \rightarrow \infty$.

A nonstochastic quantity, on the other hand, has these characterstics:

a. For given conditions its value can, in principle, be predicted by calculation.
b. It is, in general, a "point function" defined for infinitesimal volumes; hence it is a continuous and differentiable function of space and time, and one may speak of its spatial gradient and time rate of change. In accordance with common usage in physics, the argument of a legitimate differential quotient may always be assumed to be a nonstochastic quantity.

*Further discussion of stochastic vs. nonstochastic physical quantities will be found in ICRU (1971) and ICRU (1980).

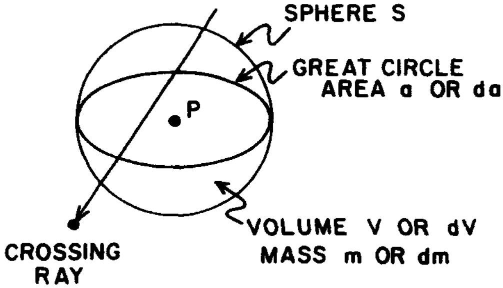

FIGURE 1.1. Characterizing the radiation field at a point *P* in terms of the radiation traversing the spherical surface *S*.

c. Its value is equal to, or based upon, the *expectation value* of a related stochastic quantity, if one exists. Although nonstochastic quantities in general need not be related to stochastic quantities, they are so related in the context of ionizing radiation.

It can be seen from these considerations that the volume of the imaginary sphere surrounding point P in Fig. 1.1 may be small but must be *finite* if we are dealing with stochastic quantities. It may be infinitesimal (dV) in reference to nonstochastic quantities. Likewise the great-circle area (da) and contained mass (dm) for the sphere, as well as the irradiation time (dt), may be expressed as infinitesimals in dealing with nonstochastic quantities. Since the most common and useful quantities for describing ionizing radiation fields and their interactions with matter are all nonstochastic, we will defer further discussion of stochastic quantities (except when leading to nonstochastic quantities) until a later chapter (16) dealing with *microdosimetry*, that is, the determination of energy spent in small but finite volumes. Microdosimetry is of particular interest in relation to biological-cell damage.

In general one can assume that a "constant" radiation field is strictly random with respect to how many rays arrive at a given point per unit area and time interval. It can be shown (e.g., see Beers, 1953) that the number of rays observed in repetitions of the measurement (assuming a fixed detection efficiency and time interval, and no systematic change of the field vs. time) will follow a Poisson distribution. For large numbers of events this may be approximated by the normal (Gaussian) distribution. If N_e is the expectation value of the number of rays detected per measurement, the standard deviation of a single random measurement N relative to N_e is equal to

$$\sigma = \sqrt{N_e} \cong \sqrt{N} \tag{1.2a}$$

and the corresponding percentage standard deviation is

$$S = \frac{100\sigma}{N_e} = \frac{100}{\sqrt{N_e}} \cong \frac{100}{\sqrt{N}} \tag{1.2b}$$

That is, a single measurement would have a 68.3% chance of lying within $\pm\sigma$

of the expectation value N_e, where σ is given by Eq. (1.2a), if the fluctuations are due to the stochastic nature of the field itself. Likewise N would have a 95.5% chance of lying within $\pm 2\sigma$ of N_e, or a 99.7% chance within $\pm 3\sigma$.

The approximation of N_e by the mean value $\overline{N}$ in Eqs. (1.2a,b) is necessary because N_e is unknown but can be approached as closely as desired by the mean value $\overline{N}$ of n measurements, i.e., $\overline{N} \rightarrow N_e$ as $n \rightarrow \infty$. It is useful to know how closely $\overline{N}$ is likely to approximate N_e for a given number of measurements n. This information is conveyed by the standard deviation of the mean value $\overline{N}$ relative to N_e:

$$\sigma' = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{N_e}{n}} \cong \sqrt{\frac{\overline{N}}{n}} \tag{1.3a}$$

and the corresponding percentage standard deviation is

$$S' = \frac{100\sigma'}{N_e} = \frac{100}{\sqrt{nN_e}} \cong \frac{100}{\sqrt{n\overline{N}}} = \frac{100}{\sqrt{N_T}} \tag{1.3b}$$

where $N_T = n\overline{N}$ is the total number of rays detected in all n measurements combined. $\overline{N}$ will have a 68.3% chance of lying within $\pm\sigma'$ of N_e. Notice in Eq. (1.3b) that it makes no difference how many measurements (n) are made in acquiring a given total count N_T, and thus a given value of S'.

It is important to emphasize that the foregoing statements of standard deviation in Eqs. (1.2) and (1.3) are based exclusively upon the stochastic nature of radiation fields, not taking account of instrumental or other experimental fluctuations. Thus one should expect to observe experimentally greater standard deviations than these, but never smaller. An estimate of the *precision* (i.e., proximity to N_e) of any single random measurement N made by a radiation detector should be determined from the data of n such measurements by means of the equation:

$$\sigma \cong \left[\frac{1}{n-1}\sum_{i=1}^{n}(N_i - \overline{N})^2\right]^{1/2} \tag{1.4a}$$

instead of Eq. (1.2a). Here N_i is the value obtained in the ith measurement, and $\overline{N} = (\Sigma N_i)/n$.

An estimate of the precision of the mean value $\overline{N}$ of n measurements should likewise be obtained from the experimental data by

$$\sigma' \cong \left[\frac{1}{n(n-1)}\sum_{i=1}^{n}(N_i - \overline{N})^2\right]^{1/2} \tag{1.4b}$$

in place of Eq. (1.3a), since $\sigma' = \sigma/\sqrt{n}$.

It should also be pointed out that the expectation value N_e of the measurements is not necessarily the physically *correct* value, and in fact will not be if the measuring instrument is improperly calibrated or is otherwise biased. N_e is merely the value of $\overline{N}$ approached as $n \rightarrow \infty$.

An example will illustrate the meaning of Eq. 1.3a:

Example 1.1. A γ-ray detector having 100% counting efficiency is positioned in a constant field, making 10 measurements of equal duration, $\Delta t = 100$ s (exactly). The average number of rays detected ("counts") per measurement is 1.00×10^5. What is the mean value of the count rate, including a statement of its precision (i.e., standard deviation)?

In Eq. (1.3a) $\overline{N} = 1.00 \times 10^5$ counts, $n = 10$ measurements, and so

$$\sigma' \cong \sqrt{\frac{\overline{N}}{n}} = \sqrt{\frac{1 \times 10^5}{10}} = 10^2 \text{ counts}$$

Thus the count rate is:

$$\frac{\overline{N}}{\Delta t} = \frac{1.00 \times 10^5 \pm 10^2 \text{ counts}}{100\ s}$$

$$= 1.00 \times 10^3 \pm 1 \text{ c/s} \qquad \text{(S.D.)}$$

This standard deviation is due entirely to the stochastic nature of the field, since the detector counts every incident ray.

B. Simple Description of Radiation Fields by Nonstochastic Quantities

1. FLUENCE

Referring to Fig. 1.1, let N_e be the expectation value of the number of rays striking a finite sphere surrounding point P during a time interval extending from an arbitrary starting time t_0 to a later time t. If the sphere is reduced to an infinitesimal at P with a great-circle area of da, we may define a quantity called the *fluence*, Φ, as the quotient of the differential of N_e by da:

$$\Phi = \frac{dN_e}{da} \tag{1.5}$$

which is usually expressed in units of m^{-2} or cm^{-2}.

2. FLUX DENSITY (OR FLUENCE RATE)

Φ may be defined by (1.5) for all values of t through the interval from $t = t_0$ (for which $\Phi = 0$) to $t = t_{max}$ (for which $\Phi = \Phi_{max}$). Then at any time t within the interval we may define the *flux density* or *fluence rate* at P as

$$\varphi = \frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{dN_e}{da}\right) \tag{1.6}$$

where $d\Phi$ is the increment of fluence during the infinitesimal time interval dt at time t, and the usual units of flux density are $m^{-2}\ s^{-1}$ or $cm^{-2}\ s^{-1}$.

Since the flux density φ may be defined by means of Eq. (1.6) for all values of t, we may thereby determine the function $\varphi(t)$, and express the fluence at P for the time interval from t_0 to t_1 by the definite integral

$$\Phi(t_0, t_1) = \int_{t_0}^{t_1} \varphi(t)\, dt \tag{1.7}$$

For the case of a time-independent field, $\varphi(t)$ is constant and Eq. (1.7) simplifies to

$$\Phi(t_0, t_1) = \varphi \cdot (t_1 - t_0) = \varphi\, \Delta t \tag{1.8}$$

It should be noted that φ and Φ express the sum of rays incident from all directions, and irrespective of their quantum or kinetic energies, thereby providing a bare minimum of useful information about the field. However, different types of rays are usually not lumped together; that is, photons, neutrons, and different kinds of charged particles are measured and accounted for separately as far as possible, since their interactions with matter are fundamentally different.

3. ENERGY FLUENCE

The simplest field-descriptive quantity which takes into account the energies of the individual rays is the *energy fluence* Ψ, for which the energies of all the rays are summed.

Let R be the expectation value of the total energy (exclusive of rest-mass energy) carried by all the N_e rays striking a finite sphere surrounding point P (see Fig. 1.1) during a time interval extending from an arbitrary starting time t_0 to a later time t*. If the sphere is reduced to an infinitesimal at P with a great-circle area of da, we may define a quantity called the *energy fluence*, Ψ, as the quotient of the differential of R by da:

$$\Psi = \frac{dR}{da} \tag{1.9}$$

which is usually expressed in units of J m^{-2} or erg cm^{-2}.

For the special case where only a single energy E of rays is present, Eqs. (1.5) and (1.9) are related by

$$R = EN_e \tag{1.9a}$$

and

$$\Psi = E\Phi \tag{1.9b}$$

Individual particle and photon energies are ordinarily given in MeV or keV, which is the kinetic energy acquired by a singly charged particle in falling through a potential difference of one million or one thousand volts, respectively. Energies in MeV

*ICRU (1980) calls R the *radiant energy*, and defines it as "the energy of particles (excluding rest energy) emitted, transferred, or received."

can be converted into ergs and joules through the following statements of equivalence:

$$\begin{aligned} 1\ \text{MeV} &= 1.602 \times 10^{-6}\ \text{erg} = 1.602 \times 10^{-13}\ \text{J} \\ 1\ \text{erg} &= 10^{-7}\ \text{J} \qquad\qquad = 6.24 \times 10^{5}\ \text{MeV} \\ 1\ \text{J} &= 6.24 \times 10^{12}\ \text{MeV} = 10^{7}\ \text{erg} \end{aligned} \tag{1.10}$$

4. ENERGY FLUX DENSITY (OR ENERGY FLUENCE RATE)

Ψ may be defined by Eq. (1.9) for all values of t throughout the interval from $t = t_0$ (for which $\Psi = 0$) to $t = t_{max}$ (for which $\Psi = \Psi_{max}$). Then at any time t within the interval we may define the *energy flux density* or *energy fluence rate* at P as:

$$\psi = \frac{d\Psi}{dt} = \frac{d}{dt}\left(\frac{dR}{da}\right) \tag{1.11}$$

where $d\Psi$ is the increment of energy fluence during the infinitesimal time interval dt at time t, and the usual units of energy flux density are J m^{-2} s^{-1} or erg cm^{-2} s^{-1}.

By identical arguments to those employed in deriving Eqs. (1.7) and (1.8), one may write the following corresponding relations for Ψ:

$$\Psi(t_0, t_1) = \int_{t_0}^{t_1} \psi(t)\, dt \tag{1.12}$$

and for constant $\psi(t)$,

$$\Psi(t_0, t_1) = \psi \cdot (t_1 - t_0) = \psi\, \Delta t \tag{1.13}$$

For monoenergetic rays of energy E the energy flux density ψ may be related to the flux density φ by an equation similar to (1.9b):

$$\psi = E\varphi \tag{1.13a}$$

C. Differential Distributions vs. Energy and Angle of Incidence

The quantities introduced in Section III.B are widely useful in practical applications of ionizing radiation, but for some purposes are lacking in sufficient detail. Most radiation interactions are dependent upon the energy of the ray as well as its type, and the sensitivity of radiation detectors typically depends on the direction of incidence of the rays striking it. Thus one sometimes needs a more complete description of the field.

In principle one could measure the flux density at any time t and point P as a function of the kinetic or quantum energy E and of the polar angles of incidence θ and β (see Fig. 1.2), thus obtaining the *differential flux density*

$$\varphi'(\theta, \beta, E) \tag{1.14}$$

typically expressed in units of m^{-2} s^{-1} sr^{-1} keV^{-1}.

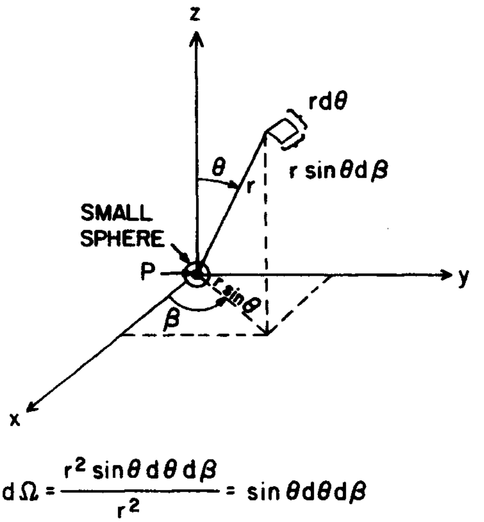

FIGURE 1.2. Polar coordinates. The element of solid angle is $d\Omega$.

Instead of the flux density distribution one could have chosen the distribution of energy flux density, or (for a given time period) the fluence or energy fluence, expressed in the proper units. The following discussion of flux density distributions can be applied to these other quantities as well.

Since the element of solid angle is $d\Omega = \sin\theta\, d\beta\, d\theta$, as shown in Fig. 1.2, it can be seen that the number of rays per unit time having energies between E and $E + dE$ which pass through the element of solid angle $d\Omega$ at the given angles θ and β before striking the small sphere at P, per unit great-circle area of the sphere, is given by

$$\varphi'(\theta, \beta, E)\, d\Omega\, dE \tag{1.15}$$

typically expressed in $m^{-2}\, s^{-1}$ or $cm^{-2}\, s^{-1}$. Integrating this quantity over all angles and energies will of course give the flux density φ:

$$\varphi = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \int_{E=0}^{E_{max}} \varphi'(\theta, \beta, E) \sin\theta\, d\theta\, d\beta\, dE \tag{1.16}$$

also in $m^{-2}\, s^{-1}$ or $cm^{-2}\, s^{-1}$.

1. ENERGY SPECTRA

Simpler, more useful differential distributions of flux density, fluence, energy flux density, or energy fluence are those which are functions of only one of the variables θ, β, or E. When E is the chosen variable, the resulting differential distribution is called the *energy spectrum* of the quantity. For example the energy spectrum of the flux density summed over all directions is written as $\varphi'(E)$, in typical units of $m^{-2}\, s^{-1}\, keV^{-1}$ or $cm^{-2}\, s^{-1}\, keV^{-1}$:

$$\varphi'(E) = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \varphi'(\theta, \beta, E) \sin\theta \, d\theta \, d\beta \tag{1.17}$$

Integration of $\varphi'(E)$ over all energies of the rays present then gives the flux density:

$$\varphi = \int_0^{E_{max}} \varphi'(E) \, dE \tag{1.18}$$

To illustrate such a spectrum, Fig. 1.3*a* shows how a "flat" distribution of photon flux density $\varphi'(E)$ would be plotted as the ordinate vs. the quantum energy as abscissa. Fig. 1.3*b* shows the corresponding spectrum of energy flux density $\psi'(E)$, where

$$\psi'(E) = E\varphi'(E) \tag{1.19}$$

That is, the ordinates in Fig. 1.3*b* are E times those in 1.3*a*. The unit ordinarily used for the factor E in Eq. (1.19) is the erg or joule, so that $\psi'(E)$ is expressed in J m^{-2} s^{-1} keV^{-1} or erg cm^{-2} s^{-1} keV^{-1}. These units convey the concept intended more clearly than would be the case if the factor E were chosen also be in keV, thus allowing cancellation of the energy units and leaving only m^{-2} s^{-1}. The joule (preferably) and the erg are the units commonly employed in describing gross energy transport in radiological physics [see Eq. (1.10)].

An equation corresponding to (1.18) can also be written for ψ:

$$\psi = \int_{E=0}^{E_{max}} \psi'(E) \, dE = \int_0^{E_{max}} E\varphi'(E) \, dE \tag{1.20}$$

In carrying out this integration in closed form it will be necessary for E to be in the same units throughout (e.g., keV), contrary to the immediately foregoing comments. The result will then be in keV/(area) (time), which can be converted to other energy units by Eq. (1.10). For numerical integration of (1.20), one may employ $\psi'(E)$ in J m^{-2} s^{-1} keV^{-1} (or erg cm^{-2} s^{-1} keV^{-1}) and still use limits and energy intervals dE expressed in keV.

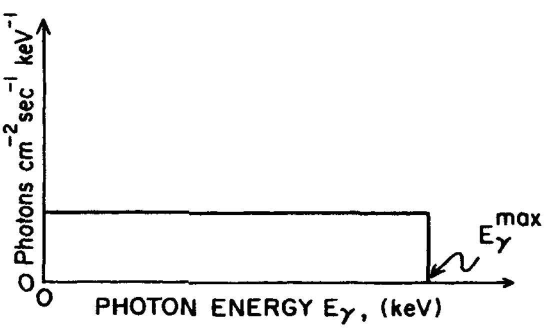

FIGURE 1.3*a*. A flat spectrum of photon flux density $\varphi'(E)$.

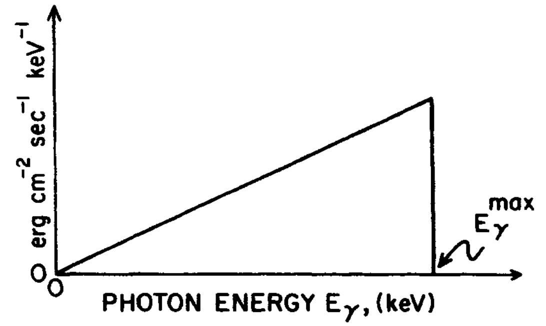

FIGURE 1.3*b*. Spectrum of energy flux density $\psi'(E)$ corresponding to Fig. 1.3*a*.

2. ANGULAR DISTRIBUTIONS

If the field is symmetrical with respect to the vertical (z) axis shown in Fig. 1.2, it will be convenient to describe it in terms of the differential distribution of, say, the flux density as a function of the polar angle θ only. This distribution per unit polar angle is given by

$$\varphi'(\theta) = \int_{\beta=0}^{2\pi}\int_{E=0}^{E_{\max}} \varphi'(\theta, \beta, E)\sin\theta\, d\beta\, dE \tag{1.21}$$

so that the flux-density component consisting of the particles of all energies arriving at P through the annulus lying between the two polar angles $\theta = \theta_1$ and θ_2 would be

$$\varphi(\theta_1, \theta_2) = \int_{\theta_1}^{\theta_2} \varphi'(\theta)\, d\theta \tag{1.22}$$

where $\varphi'(\theta)$ can be expressed in m^{-2} s^{-1} radian^{-1}, for example. For θ-limits of 0 and π, this integral of course gives φ.

Alternatively one can obtain the differential distribution of flux density per unit *solid angle*, for particles of all energies, as

$$\varphi'(\theta, \beta) = \int_{E=0}^{E_{\max}} \varphi'(\theta, \beta, E)\, dE \tag{1.23}$$

in typical units of m^{-2} s^{-1} sr^{-1}. This may be integrated over all directions to again obtain the total flux density:

$$\varphi = \int_{\theta=0}^{\pi}\int_{\beta=0}^{2\pi} \varphi'(\theta, \beta)\sin\theta\, d\theta\, d\beta \tag{1.24}$$

For a field that is symmetrical about the z-axis, $\varphi'(\theta, \beta)$ is independent of β; hence Eq. (1.24) can be integrated over all β-values to obtain

$$\varphi = 2\pi \int_{\theta=0}^{\pi} \varphi'(\theta, \beta) \sin\theta \, d\theta \tag{1.25}$$

Comparing this equation with Eq. (1.22) over the limits $\theta = 0$ to π reveals that, for the case of z-axis symmetry, $\varphi'(\theta)$ is related to $\varphi'(\theta, \beta)$ by

$$\varphi'(\theta) = (2\pi \sin\theta)\, \varphi'(\theta, \beta), \tag{1.26}$$

where $\varphi'(\theta)$ has the units $m^{-2}\, s^{-1}\, radian^{-1}$ and $\varphi'(\theta, \beta)$ is given in $m^{-2}\, s^{-1}\, sr^{-1}$. Figure 1.4 illustrates this relationship for the case of a completely isotropic field (solid curves), and for the case where $\varphi'(\theta, \beta)$ is still β-independent but varies as some function of θ(dashed curves). $\varphi'(\theta, \beta)$ is arbitrarily taken as $(1 - \theta/\pi)$ in the latter case shown.

Sometimes one is interested in expressing the flux density of particles of all energies as a function only of the azimuthal angle β. Then $\varphi'(\theta, \beta)$ from Eq. (1.23) may be used, where one usually sets $\theta = \pi/2$.

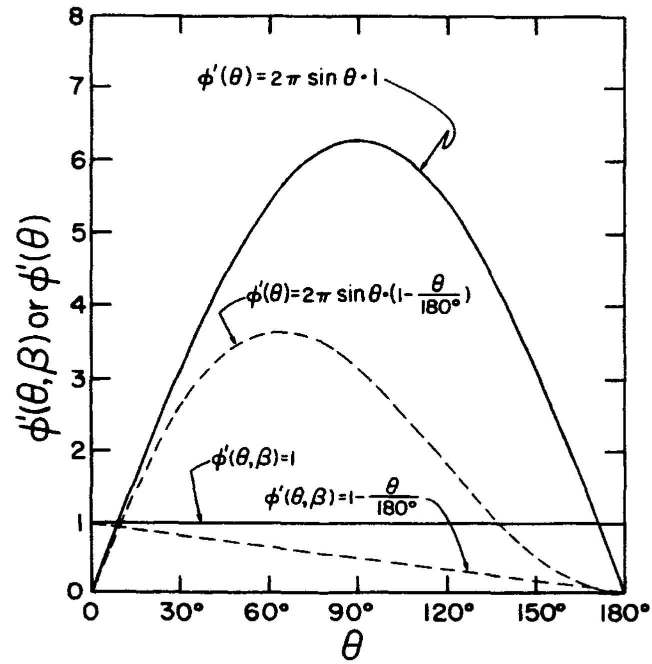

FIGURE 1.4. Isotropic radiation field expressed in terms of its flux-density distribution per unit solid angle, $\varphi'(\theta, \beta)$ = constant $\equiv 1\ m^{-2}\, s^{-1}\, sr^{-1}$ (lower solid curve). The same field is also shown in terms of its distribution per unit polar angle, $\varphi'(\theta)$, in $m^{-2}\, s^{-1}\, radian^{-1}$ (upper solid curve). These two curves are related by the factor $2\pi \sin\theta$, which is also true if $\varphi'(\theta, \beta)$ is a function of θ only [e.g., see dashed curves for $\varphi'(\theta, \beta) = 1 - (\theta/180°)$].

D. An Alternative Definition of Fluence

Chilton (1978, 1979) has proven the validity of an alternative definition of fluence, namely:

> The fluence at a point P is numerically equal to the expectation value of the sum of the particle track lengths (assumed to be straight) that occur in an infinitesimal volume dV at P, divided by dV.

This statement was shown to be true for nonisotropic as well as isotropic fields, irrespective of the shape of the volume. Thus one need not require a spherical volume to define fluence in this way. Moreover this definition lends itself to dosimetry calculations by the Monte Carlo method.

E. Planar Fluence

Planar fluence is the number of particles crossing a fixed plane in either direction (i.e., summed by scalar addition) per unit area of the plane. The name "planar fluence" was given to it by Roesch and Attix (1968), who also defined a vector-sum quantity corresponding to the planar flux density that they called the *net flow*, that is, the number of particles per unit time passing through unit area of the plane in one sense (say side A to side B) *minus* those going the other way ($B \rightarrow A$). This quantity is of little dosimetric relevance, however. Although vectorial methods are convenient for field calculations, as shown by Rossi and Roesch (1962) and Brahme (1981), radiation dosimetry finally requires scalar, not vector, addition of the effects of individual particles.

The concept of net flow was first put forward in the context of radiological physics by Whyte (1959). He dealt with the flow of energy carried by particles, and applied the name "plane intensity" to the vector sum of the energy flowing through a fixed plane. Whyte's illustrative diagram is reproduced in Fig. 1.5, which will be used here to discuss fluence vs. planar fluence.

A plane homogeneous beam of radiation is shown perpendicularly incident upon a flat scattering (but not absorbing) foil. All particles are shown for simplicity being scattered through the same angle θ, at any azimuthal angle β. A spherical and a flat detector of equal cross-sectional area are shown positioned above and below the foil. The flat detector is oriented parallel to the foil, and thus is perpendicular to the beam of incident radiation. The number of incident particles striking each detector above the foil is clearly the same, and the planar fluence with respect to the plane of the flat detector is identical to the fluence in the same field. This can only be true in a plane-parallel beam, orthogonal to the flat detector, as shown.

The number of scattered particles striking the spherical detector below the foil is $|1/\cos\theta|$ times the number striking the flat detector, which in turn is the same as the number it received above the foil. Thus the fluence is $|1/\cos\theta|$ times the planar fluence. This increase in fluence contributes to an effect sometimes seen in broad-beam geometry, in which the fluence behind an attenuating layer can be greater than that incident (see Chapter 3, Section V).

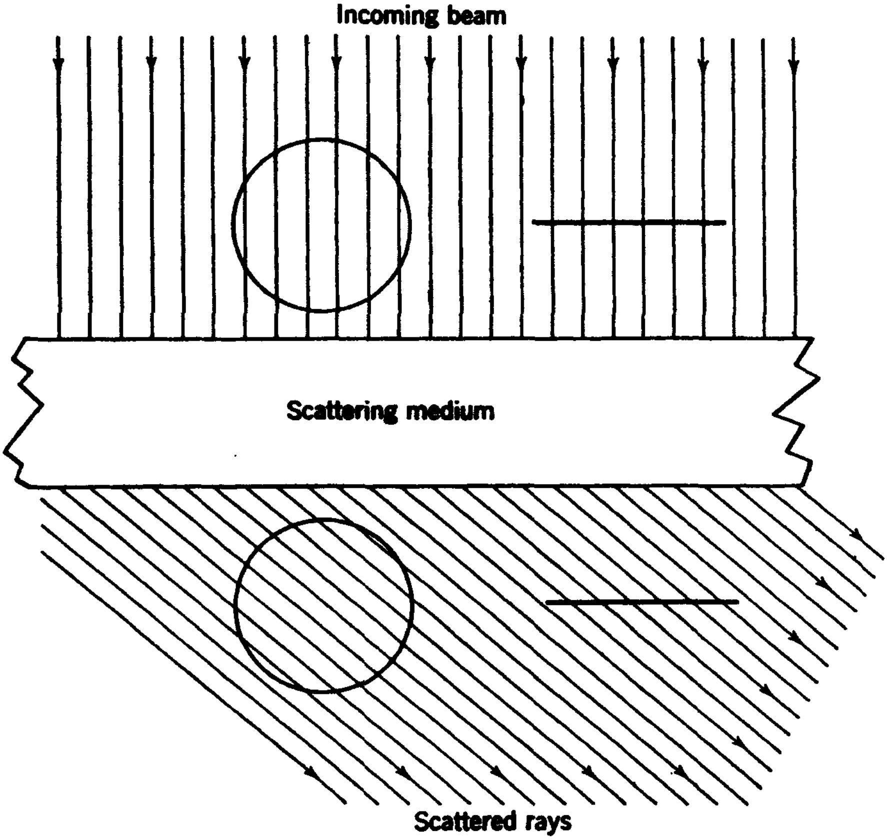

FIGURE 1.5. Particles scattered through an angle θ in a nonabsorbing foil, illustrating effect on fluence vs. planar fluence. (After Whyte, 1959.)

The effect of radiation striking a detector depends on the penetrating power of the radiation. Consider the two limiting cases in which: (a) the radiation penetrates straight through both detectors shown in Fig. 1.5, and (b) the radiation is stopped and absorbed in both detectors. For both cases we will take the response of the detector to be proportional to the energy imparted in it.

For case (a) we will also assume that the energy imparted is approximately proportional to the total track length of the rays crossing the detector, or to the fluence according to the Chilton definition. This assumption is by no means proven at this point, but it is reasonably good for homogeneous radiation crossing a small, easily penetrated detector. The spherical detector in Fig. 1.5 will read more below the foil in proportion to the number of rays striking it, which is $|1/\cos\theta|$ times the number striking it above the foil. The average length of the paths in the sphere is obviously the same above and below. The number of rays striking the flat detector is the same above and below the foil, but the length of each track within the detector is $|1/\cos$