Dyslexia, Dyspraxia and Mathematics

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Emerson House, London

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Foreword

Education is never still. There are constant changes and developments. Some of these enhance our knowledge of how children learn and thus how they can be taught most effectively. Some are merely cosmetic where the only increase in output is in administration. This book takes a proud place in the first category.

Our awareness of the diversity of special needs has increased greatly over the past thirty years. For example the term 'specific learning difficulties' was once promoted as the preferred term for 'dyslexia'. Now we are aware of a cluster of specific learning difficulties, including dyspraxia.

Awareness of a learning difficulty is a good start, but it can result in stereotypical concepts and inadequate and inappropriate interventions. What Dorian Yeo has done in this book is to extend awareness to understanding and has then set the understanding of the *individual* within sound, clearly and thoroughly explained underlying principles.

Before the publication of this book there was a great need for material for the younger learner and for the dyspraxic learner. It is fascinating to see the comparisons between the problems experienced by dyslexic learners and dyspraxic learners. There seem to be more similarities than differences, which will not come as a surprise to those who work in special education. As important as this observation is, the realisation that good intervention for special needs is good intervention for all learners is even more important. Few learners (if any) are perfect, so this too should not be a surprise.

This book is about good practice. To paraphrase Professor Tim Miles, this good practice will help all learners, but it is an essential for dyspraxic and dyslexic learners.

Dorian Yeo's book is written from deep personal understanding and knowledge, but not from knowledge built on a sample of one, but of many similar yet diverse pupils. It combines a comprehensive explanation of the difficulties young children experience when learning numeracy with many practical, structured and developmental ideas for teaching. It has been a privilege to see Dorian's vision become reality.

Steve Chinn
August 2002

Preface

Emerson House is a small and intensive specialist teaching and learning centre which caters for the core learning needs of dyslexic and dyspraxic primary school children. When I established the maths department at the rapidly growing Emerson House some years ago, I already knew that many dyslexic children did not learn maths as easily as the majority of 'ordinary' children did. I had also discovered, through experience, that access to concrete materials could make a difference to the performance of dyslexic children. In the early days, however, I assumed that children with specific learning difficulties simply needed more practice – with concrete support – in order to make progress in learning the aspects of maths which they found hard. From working with our children and our teachers in a questioning way, I discovered that 'overlearning', however patiently orchestrated, was often not enough. Inspired by Steve Chinn and Richard Ashcroft's work with secondary school pupils, I realized that we needed to know more about how young children make sense of numbers and why some children – and dyslexic and dyspraxic children, in particular – can find the early stages of working with numbers so difficult. As we began to change aspects of how we taught maths at Emerson House, we realized that our most important task was to set out to make the foundations of number-work as simple, clear and easily understood as possible. In this book I have set out to describe the teaching ideas which have made a difference to the happiness, confidence, progress and attitude towards maths of the children whom we have taught.

I would like to thank Jane Emerson for the support and encouragement which she has always given me.

Dedication

For my husband, Dudley, and for my children, Lisa, Claire and Russell. Also for my sister, Kay, who, like me, survived; and for my brother, mother and father who did not.

PART I

Definitions and Premises

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CHAPTER 1

Background information

Introduction

This is a book which sets out to explore how primary school dyslexic and dyspraxic children with varying degrees of maths learning difficulties understand and learn maths. It discusses a number of important ideas about some of the cognitive features which seem to underlie general maths learning difficulties or which may underlie difficulties learning specific aspects of maths, such as the *times tables* facts. It outlines the ways in which children usually learn the foundation aspects of maths and considers the special cognitive needs of dyslexic and dyspraxic children in this context. It aims to offer practical support and detailed teaching suggestions to teachers, tutors and parents who wish to help dyslexic or dyspraxic children make real and sustained progress in learning maths.

The book has grown out of the experience of teaching maths to primary school dyslexic and dyspraxic children, aged from about 6½ years upwards. From the outset the shape of this experience has been profoundly influenced by the work of Steve Chinn and by Steve Chinn and Richard Ashcroft's seminal and forward-looking book, *Mathematics for Dyslexics: A Teaching Handbook* (1998). Above all, the experience has been driven by the conviction, based on experience, and directly expressed in *Mathematics for Dyslexics*, that to teach maths well to children with specific learning difficulties, 'a different attitude and approach is needed' (Chinn and Ashcroft, 1998, p. 1).

In the challenging and ongoing process of further developing and refining 'a different attitude and approach' which aims to meet the needs of younger primary school dyslexic and dyspraxic children, there have been a number of other important sources and influences. It has, of course, been possible to draw on the available research into 'dyslexia specific' maths difficulties and, in particular, into their difficulties with memorized *times tables* facts. There are also valuable teaching-based accounts of the ways in which many of the underlying cognitive difficulties associated with dyslexia seem to affect the maths learning abilities of dyslexic children. Such accounts include those of Chinn and Ashcroft (1998), Henderson (1989; 1998), Henderson and Miles (2001) and Miles and Miles (1992). However, as Steve Chinn frequently points out, there is a disappointingly limited body of research and a relatively limited range of literature exploring themes related to dyslexia and maths learning. In particular, there is very little consideration of the typical learning profiles and particular learning needs of young primary school dyslexic and dyspraxic children.

In recent years, on the other hand, there has been an explosion of research into how 'ordinary' young children make sense of numbers. Many studies set out to explore the ways in which children come to learn about, and make progress in, the crucial foundation stages of number-work. These studies have practical implications for how these 'building block' areas of maths are best taught. Furthermore, a number of contemporary researchers are interested in understanding the maths behaviours of the small numbers of children who can be found in any maths classroom who have difficulty making sense of numbers and who fail to make progress in number-work from the very earliest stages of mathematics. This contemporary research provides many illuminating insights into the maths learning profiles of primary school dyslexic and dyspraxic children. In particular, as we will see, the work of Karen Fuson and her colleagues in the US, Ian Thompson, Eddie Gray and Judith Anghileri, in the UK, and the work of the contemporary generation of Realistic Education proponents in The Netherlands have shaped many aspects of the 'attitude and approach' to teaching maths which are described in the book.

More general but illuminating books by Brian Butterworth (1999) and Stanislas Dehaene (1997) – both of which set out to explore the anthropological and biological bases of our knowledge of numbers – have also influenced some of the key discussions. In particular, I am indebted to Brian Butterworth's very clear account of the structure of the number system and to Dehaene's suggestive ideas about the *times tables*. I have also tried to make a layperson's sense of their neuroscientific contribution towards understanding why and how we know about numbers in the first place and make some brief and tentative suggestions about what their studies and accounts of 'maths in the brain' seems to suggest about maths learning difficulties.

Which pupils does the book cater for?

Dyslexia, Dyspraxia and Mathematics explores ways of helping primary school dyslexic and dyspraxic children acquire a sound foundation in all of the key numeracy aspects of maths. Although many dyspraxic children also have difficulty with the non-number, spatial aspects of maths, these difficulties have not been addressed in this book.

In the last decade, or so, it has been increasingly widely accepted that a substantial proportion of dyslexic children have at least some difficulties learning the basic number aspects of maths. In recent years, as more children are diagnosed as belonging to the dyspraxic side of the specific learning difficulties spectrum, many teachers, educational psychologists and parents are finding that a very substantial proportion of young dyspraxic children have difficulties with number-work, too.

It is often noted that it is hard to generalize about the maths learning abilities and difficulties of dyslexic children. As Chinn and Ashcroft write,

> many dyslexics have difficulty in at least some aspects of mathematics, but this is not necessarily in all aspects of mathematics. Indeed, some dyslexics are gifted problem solvers, despite persisting difficulties in, for example, rote learning of facts. (1998, p. 14)

Some researchers, for example Steeves (1983) and Miles and Miles (1992) have found that a number of dyslexics are broadly gifted in most aspects of maths and informal reports from teachers and parents seem to confirm Steeves's and Miles and Miles's finding.

Unfortunately there appears to be no detailed published research on the maths abilities of children who have been formally diagnosed as dyspraxic. Standard ability measures used by educational psychologists show that dyspraxic children are often weak at maths. Teachers report that a great many of the dyspraxic children in their classrooms have difficulties with the numeracy aspects of maths. Diagnostic assessments and teaching experience show that dyspraxic children with high verbal scores and with long-term and working-memory strengths frequently do well in the routine, procedural aspects of maths. However teachers report that a great many of the dyspraxic children whom they teach have severe problems in the very earliest stages of maths and that most dyspraxic children have serious word-problem-solving, 'number-puzzle'-solving, and pattern-solving weaknesses which persist throughout their primary school careers.

Although it is hard to generalize about the maths abilities of dyslexic children, as we have noted, it is well documented and widely acknowledged that dyslexic children who show some ability in the numeracy aspects of maths nevertheless often have marked difficulty with two of the foundation aspects of number-work: dyslexic children typically have difficulty remembering exact maths facts, such as the *times tables* facts, and (like many dyspraxic children) dyslexic children also have difficulty with the *place value* conventions of the written number system. This book addresses these seemingly 'dyslexia-typical' and 'dyspraxia-typical' weaknesses and offers practical teaching suggestions which will help children in these areas.

While the available literature has offered a fairly clear picture of the number-related 'gaps' which can be described as 'dyslexia-typical' behaviours, it has been less widely acknowledged that some dyslexic children, and many dyspraxic children, have difficulties with number-work which are really very deep-seated and profound and which seem to go back to the very earliest stages of making sense of numbers. In fact, classroom teachers report that a sizeable proportion of dyslexic and dyspraxic children have quite severe all-round (global) maths learning difficulties. It is this group of children – the children who fail to make progress from the earliest stages of learning about numbers onwards – whom classroom teachers and parents are often most concerned about. It is also quite typically this group of children who do not seem to respond to 'ordinary' additional maths tuition and who seem to require 'specialist' understanding and help. This book sets out to examine the better understood 'typical' dyslexic and dyspraxic maths learning difficulties. It also sets out to begin charting the maths-learning profiles and apparent learning needs of the hitherto rather neglected group of dyslexic primary school maths learners who appear to have longstanding difficulties with all aspects of number-work and who can be described as children with very significant maths learning difficulties. As we will see later, these are also the children who are sometimes labelled *dyscalculic*.

Recent changes in maths teaching and a consideration of present-day maths learning situations

In broad maths educational terms this is an exciting but also potentially confusing time to be thinking about maths teaching and learning. In the wake of the explosion of research into how young children understand and learn about numbers, which was briefly mentioned above, far-reaching educational reforms have been introduced into primary school maths classrooms in many parts of the Western European world, including in the UK. The radical National Numeracy Strategy Framework was implemented in English state school classrooms in September 1999.

In general terms, many of the recent maths education reforms have been largely positive ones for dyslexic and dyspraxic maths learners. Influential contemporary maths educationalists, including Chinn and Ashcroft, have campaigned for a long time for some of the changes which have been enshrined in the Numeracy Strategy Report and in the Numeracy Strategy Framework. For instance, in reaction to the understood shortcomings of traditional maths teaching, the Numeracy Strategy, like most other newer maths teaching approaches, sets out to try and help children make genuine sense of mathematics. Instead of expecting that children simply learn facts and procedures solely *by heart*, or through rote 'drill', there is an emphasis on helping children understand logical principles, important concepts, and underlying patterns and structures. In keeping with this, there is a far greater emphasis on 'mental' mathematics, in general, and on logic-based and numeracy-friendly, informal ways of calculating.

The contemporary ideas about maths learning, together with the reforms they have inspired, have not, however, affected all primary school children in the UK in the same measure. The maths educational 'map' of what actually happens on the ground in maths classrooms is quite complex at present. While state-sector classrooms in England follow the Numeracy Strategy guidelines, the Numeracy Strategy has not been implemented in Scotland, Wales or Northern Ireland; and although Scotland and Wales have instituted their own maths teaching reforms, maths continues to be taught in quite traditional ways in Northern Ireland. Furthermore, private-sector schools in England are not bound to implement the National Numeracy Strategy.

In fact, the maths educational picture in private-sector schools in England is particularly complex and would seem to be in a state of flux at present. On one hand, it is evident that the Numeracy Strategy has had ripple effects on maths teaching in a number of private schools: for example, the teaching ideals, goals and recommendations of the Numeracy Strategy have shaped the ways that recent maths textbooks and schemes have been designed, and the Numeracy Strategy has also informed the content of the standardized maths National Curriculum Tests. Some head teachers and heads of maths departments in private schools have actively welcomed changes in maths teaching practices, and have looked favourably on the impetus towards reform – in particular, on the greater weight accorded in newer maths teaching approaches to *mental maths*. On the other hand, it is also evident that a considerable number of private schools have continued to teach maths in largely unchanged, traditional ways. Many private schools continue to use older teaching methods, textbooks and materials, and many private schools also place greater weight on the results of hitherto more traditional 'Common Entrance' maths papers (or similar papers) than they do on the reform-based goals of the National Curriculum tests.

In the context of this book, it is of course the learning needs – and therefore the teaching requirements – of dyslexic and dyspraxic children which is the paramount consideration. Although the teaching issues are extremely complex (and some of the complexities will be explored, later on) it is also important to give an overview perspective on the ways in which different approaches to maths teaching and learning can affect the ability of dyslexic and dyspraxic children to make progress in learning maths.

On one hand, as we will see, and as Steve Chinn has frequently demonstrated, the memory requirements of traditional maths approaches create broadly *unfavourable* maths learning environments for dyslexic and dyspraxic maths learners and contribute to the severe difficulties that most dyslexic and dyspraxic children experience in the majority of traditional maths classrooms. The complex memory difficulties, which are commonly associated with dyslexia and dyspraxia, and which affect maths learning, will be explored in greater detail later on. In brief, however, traditional maths approaches require that maths facts are acquired through rote learning with little emphasis on the inter-relationships between facts. A good proportion of maths learning time is devoted to memorizing standard calculation procedures in columns. Learning the standard procedures depends on a good visual memory and a very good memory for sequential sets of instructions.

On the other hand, institutionalized, progressive approaches to teaching maths, such as that embodied in the Numeracy Strategy, create potentially *favourable* environments for dyslexic and dyspraxic children to learn maths. As suggested above, most of the principles and goals which lie behind recent maths reforms are principles and goals which apply, in essence, to dyslexic and dyspraxic maths learners, too. However, the cognitive weaknesses associated with dyslexia and dyspraxia, and the severity of the weaknesses affecting individual children, also influence the degree to which dyslexic and dyspraxic children are able to make progress in mainstream maths teaching approaches – approaches which are, in the main, designed to cater for the learning needs of 'ordinary' young children. It is perhaps not surprising that the progress of dyslexic and dyspraxic children in, for example, Numeracy Strategy classrooms has been somewhat mixed to date. On the one hand teachers report that although maths fact acquisition continues to be an area of difficulty for most dyslexic children, more mathematically able dyslexic and dyspraxic are generally enjoying maths and are making good progress within the framework of the relatively flexible, pattern and logic-based approach to learning maths which characterizes the Numeracy Strategy. On the other hand, there have been more worrying reports that some children with specific learning difficulties – generally children who are found to have moderate to severe maths learning difficulties – are not faring particularly well in classrooms which are guided by the Numeracy Strategy approach. Indeed, since 1999 it has become increasingly clear that a significant number of dyslexic and dyspraxic children – together with other children who find maths learning difficult – are not making expected progress in otherwise successful state school classrooms. Many educationalists, teachers, and support teachers have recognized that if all dyslexic and dyspraxic children are to be helped to make the best possible progress in learning mathematics some teaching practices will have to be modified to take account of the number-related learning needs which dyslexic and dyspraxic children may have.

To sum up: Steve Chinn, and Steve Chinn and Richard Ashcroft, have convincingly shown that traditional maths teaching approaches do not suit the learning needs of the vast majority of dyslexic children. They have pioneered the argument, in the UK, that children with specific learning difficulties need to be taught in such as way that they are able to understand all aspects of the maths they are learning and that they need to be taught to reason effectively instead of being expected to rote learn facts and procedures. They have passionately fought for open-minded and flexible maths classroom environments. Contemporary progressive maths approaches, such as that of the Numeracy Strategy, share many of the maths teaching and learning ideals and principles for which Chinn and Ashcroft have campaigned. However overview assessments of children's progress at the primary school level shows that the particular maths learning needs of a great many dyslexic and dyspraxic children seem to require a degree of special consideration. This book is inspired by the aim of describing an understanding-based approach to teaching the numeracy aspects of primary school maths which also takes into account the special cognitive features of dyslexic and dyspraxic primary school children.

Basic definitions and important features associated with dyslexia and dyspraxia

While there can be marked cognitive and behavioural differences between *classic dyslexic* and *classic dyspraxic* learners, the distinctions are often harder to make in practice. As Madeline Portwood (2000) clearly explains, there is a very significant degree of overlap or comorbidity between different specific learning difficulties, such as dyslexia and dyspraxia. Madeline Portwood explains further that dyslexia and dyspraxia can also be comorbid with attention deficit disorders (ADD and ADHD) and Asperger's syndrome. From the point of view of this book this means that many children will have both dyslexic and dyspraxic cognitive and learning features. Dyslexia and dyspraxia also share a number of important learning-related characteristics, as we will see. This partly explains why so many classic dyspraxic learners have been diagnosed as dyslexic in the past.

Nevertheless it is important to acknowledge that *classic dyslexia* and *classic dyspraxia* are associated with some widely divergent weaknesses and strengths. The brief descriptions of dyslexia and dyspraxia, which follow, draw attention to some key differences and some of the key similarities in the cognitive profiles of dyslexic and dyspraxic children.

'Classic dyslexia'

Basic definitions of dyslexia usually centre on the difficulties which dyslexics experience in processing the symbolic aspects of language. Dyslexic children have difficulty learning to read and spell in large part because they have difficulty mapping segments of sound (phonemes) on to written symbols (graphemes). Some dyslexics have difficulties with phoneme awareness or the initial discrimination of sounds. Many dyslexic children have language acquisition, word finding or semantic (meaning-related) difficulties. Underlying cognitive weaknesses associated with dyslexia include: poor long-term verbal memory; poor working memory; poor sequencing skills and sequential memory; difficulties with auditory and/or visual perception and memory; and poor left/right discrimination. Because dyslexia is so strongly associated with difficulties to do with processing symbols, *standardized assessment profiles* which are administered by educational psychologists – for example, the Wechsler Intelligence Scales or WISC – tend to show depressed *verbal* scores in relation to overall *intelligence* and *performance* (non-verbal) stores. In terms of very broad brain function, dyslexia is often broadly characterized as a tendency towards general left-hemispheric weakness. On the other hand, the *performance* scores of many dyslexic individuals show strengths in spatial or visuo-spatial areas. Such strengths, 'which may contribute to outstanding creative skills' (L. Peer in Smythe, 2000, p. 67), are sometimes said to be the 'compensatory gift' of dyslexia. Although some *classic dyslexics* do not have exceptionally strong *performance* skills, dyslexia is associated, generally speaking, with a tendency towards relative right hemispheric strengths in the brain (Portwood, 2000).

'Classic dyspraxia'

In essence, dyspraxia is associated with motor co-ordination difficulties – often with gross and fine motor co-ordination difficulties – and with perceptual and spatial-perceptual weaknesses. According to Portwood (2000, p. 26) all dyspraxics have 'co-ordination difficulties' and the vast majority 'show significant perceptual problems' (p. 26). Additional weaknesses associated with dyspraxia include: left/right confusion; poor tactile perceptual skills; poor hand–eye co-ordination; poor working memory; poor visual memory; poor sequencing skills; poor short-term visual or auditory memory; poor verbal memory; poor memory for verbal instructions; and finger agnosia (loss of 'finger sense' or an intuitive knowledge of the fingers). Standardized assessments of dyspraxics classically show depressed *performance* scores. They also show that 'on average verbal scores are higher than performance.' (Portwood, 2000, p. 47). In terms of hemispheric dominance this means that dyspraxia can be associated with a tendency towards right hemispheric weakness or 'immaturity' and with relative left hemispheric strength. In this regard it is worth noting that classic dyspraxia is not always associated with significant difficulties with learning to read although it is often associated with significant spelling difficulties.

Some cognitive weaknesses which dyslexic and dyspraxic children commonly share and which can affect maths learning

It is, of course, vital to acknowledge that each individual dyslexic or dyspraxic child will bring 'different combinations of strengths and weaknesses' to maths and that, as we have seen, there are 'enormous variations' in maths abilities among individual children with specific learning difficulties (Chinn and Ashcroft, 1998, p. 5). However, it is also vital to be aware of what one might call 'the big picture' in the relationship between specific learning difficulties and maths learning difficulties. Before we go on to explore some of the significant ways in which dyslexic and dyspraxic learners can diverge in terms of how they process number tasks, it is important to start out by outlining some of the significant underlying learning constraints which a great many dyslexic and dyspraxic children have in common.

Poor long-term memory in maths learning

It is widely documented that dyslexic and dyspraxic children have difficulty automatizing maths facts and maths procedures – in other words, dyslexic and dyspraxic children have difficulty recalling number facts (such as subtraction facts, or multiplication facts) or the way to 'do sums'. As we will see, working memory weaknesses and sequencing difficulties contribute to many of the long-term memory difficulties which dyslexic and dyspraxic children experience in maths. In addition to this, as Miles and Miles (1992) and Chinn and Ashcroft (1998) have shown, the majority of dyslexic children find it a 'frustrating exercise' to learn verbally encoded facts (Chinn and Ashcroft, 1998, p. 68) and find it almost impossible to recall many of them 'in one'. Over and over again it is commented on that dyslexic children, and many dyspraxic children, fail to learn facts easily in the form of pure verbal associations. This has a disastrous impact on *times tables* learning, as we will see.

Poor working memory

Working memory weaknesses impact on maths learning in at least two key ways. First of all, most aspects of working with numbers, from basic counting onwards, are linear or step-by-step processes which involve holding several pieces of information in working memory at the same time. Children with learning difficulties often lose track of what they are doing, forget what the initial task was or forget the teacher's instructions. 'What was the sum again?' is a classic 'dyslexic' or 'dyspraxic' question. As Chinn and Ashcroft (1998, p. 8) explain,

> The pupil may not be able to 'hold' the visual image of the sum he is trying to solve. He may not be able to hold the sum in visual or auditory memory while he searches for a necessary number fact.

As number-work becomes more demanding, a greater number of elements need to be held in working memory at once. At the most obvious level, poor working memory affects the child's speed of thinking and calculating and, indeed, working in general in maths.

Secondly, memory weaknesses contribute to long-term memory difficulties and vice versa. The relationship between working memory and long-term memory is obviously complex, but, generally speaking, it would appear that in order for number information to enter long-term memory, working memory processes need to be relatively efficient. According to Ashcraft et al, the working memory has a limited capacity. In Eddie Gray's (1997) useful and succinct formulation, for information to enter long-term memory, in maths, it is important that the *input*, or question, and the *output*, or the answer to the question, are close together. In simple terms, poor working memory, or lengthy working memory processes, mean that number information is less likely to enter long-term memory. As we will see, sequencing difficulties contribute significantly to working memory difficulties in number-work, and ultimately – in a vicious cycle of cause and effect – to long-term memory difficulties. On the other hand, Ashcraft et al. (1996, p. 195) argue that long-term memory weaknesses (poor fact and procedure retrieval) contribute to working memory problems: long-term memory weaknesses drain processing resources or capacity from the 'executive' or managing component of the working memory in another complex cycle in which it is hard to disentangle cause and effect. In simple terms, limited working memory resources are 'drained' when children have to spend time trying to work out facts or trying to remember procedures and, once again, the ultimate outcome, or the steps of the procedure, are not remembered in the long-term.

Ashcraft et al. also maintain that maths anxiety affects the efficient functioning of working memory. We will touch on the theme of anxiety towards the end of this chapter, but one of Ashcraft et al.'s (1996, p. 193) significant suggestions is that anxiety causes 'intrusive thoughts and worry' to drain working memory resources. Hence Ashcraft et al. believe that maths anxiety affects cognitive functioning in maths tasks and therefore affects overall maths learning in a direct way. Chinn and Ashcroft (1998) and Henderson (1989, 1998) have noted that dyslexic children are often anxious about specific aspects of maths, such as the *tables* facts or division, and that some dyslexic children are anxious about maths in general. They show that many dyslexic and dyspraxic children are not confident enough to 'have a go' at answering 'challenging' maths questions: 'no attempt' errors contribute significantly to the poor scores which are commonly attained by dyslexic and dyspraxic pupils in maths assessments. Many parents of dyslexic and dyspraxic children volunteer that their children are anxious about maths. In diagnostic assessments of primary school dyslexic and dyspraxic children more than 70 per cent of the children assessed stated that they 'hated' maths.

Sequencing problems

The broad label *sequencing problems* covers a number of complexly interrelated areas:

1. *Counting and the number system* Very early on, from the beginning stages of learning how to count, children have to make sense of, and have to learn to use, complex sequences of number words. As we will see, the ability to count also involves mapping words on to sequences of objects. Remembering sequences of words and seeing patterns within these sequences is a crucial aspect of learning to understand the complex structures of the number system. Many dyslexic and dyspraxic children learn to count later than their peers and fail to understand the structures of the number system. In consequence they have difficulty *decoding* large numbers and have problems solving mental and written large number calculations.
2. *Counting in number-work* As we will also see, much of early number-work is bound up with quite lengthy sequences of counting, for example, in *counting on* in addition, and in *counting back* in subtraction. Difficulty managing counting sequences, and especially 'backwards' sequences, impacts on working memory efficiency and on the automatization of facts.
3. *Sequences of instructions* It is widely noted that many dyslexic and dyspraxic children have difficulty remembering sequences of verbal instructions. All larger number calculations involve completing a sequence of steps. The familiar standard calculation methods are usually taught 'procedurally' or as a series of verbal instructions – 'first, you do this, next you do this, next you do this, then you ...' Standard methods are consequently particularly difficult for dyslexic and dyspraxic children to learn. The newer mental calculation methods are generally relatively easy to understand and do not require to be taught in 'recipe-like' ways. However, a number of teachers do, in fact, resort to teaching mental methods as rote-learned verbal routines: this happens, in particular, when teachers wish children to learn a number of different mental methods for specific operations.

Directional confusion

Although *directional confusion* can simply mean that young children muddle written digits, for example '2' and '6', directional difficulties usually become particularly significant when two-digit numbers need to be read, or decoded, and written or encoded. As we will see, the fact that the number system is structurally different in the crucial second decade between 10 and 20 means that many children become confused about which digit in a two-digit number to say or write first. These 'normal' difficulties are compounded if children confuse direction or 'position.' Directional confusion can mean that children who know that the larger value is usually read or written first may sometimes still be confused about what 'first' means in positional terms – in 'left/right' terminology a child may suddenly flounder as to whether 'first' means left or right.

Directional confusion can have particularly devastating consequences if children are taught column-based methods of multi-digit calculation from the outset. As we will see, column-based methods of addition and subtraction begin from the right whereas numbers, words and sentences are read from the left. As Chinn and Ashcroft point out, the starting point for the standard division algorithm, which is on the left, can upset children's hard-won and overlearned 'right-to-left in calculation' response. In 'borrowing' in standard subtraction procedures, children have to move left and right and left in extremely taxing ways. Finally, the 'crossover' directional demands of formal long multiplication and the across-and-step-down demands of formal long division methods contribute to the difficulties which many dyslexic and dyspraxic children experience in trying to reproduce the standard ways of executing calculations.

Speed of working in oral and written work

Madeline Portwood (2000) suggests that the neurological immaturities ('wiring immaturities'), which may contribute to dyslexia and dyspraxia, mean that children with learning difficulties tend to be slow to process incoming information. Even when maths information has been lodged in long-term memory, many dyslexic and dyspraxic children take longer to access this stored information. It is often noted that an over-emphasis on requiring dyslexic or dyspraxic children to give quick answers to maths facts questions, or to figure out mental calculations rapidly, has the effect of undermining the dyslexic or dyspraxic child's ability to think. Of course working memory difficulties, attention deficits and anxiety all compound delays in processing and retrieving information. As Chinn and Ashcroft argue, this often has the further result that children with learning difficulties complete written work more slowly and complete less work than their peers. This, in turn, can mean that children may not reach more challenging examples in written exercises or may fail to reach, and therefore have the opportunity to work through, certain exercises altogether.

Poor ability to generalize in mathematics: a weak basic number-concept

In an early analysis of the maths profiles of dyslexic learners, Joffe (1983b) observed that many dyslexic pupils do not easily generalize the knowledge they have acquired in number-work. In supporting this statement, Chinn and Ashcroft (1992, p. 98) say, 'in our experience of teaching dyslexics we have observed another handicapping factor: a poor ability to generalize and classify facts and rules in mathematics'. They go on to say that, in their experience, many of their secondary school students view maths 'as an amorphous, disjointed mixture of facts, rules and methods. Although they can understand these parts in isolation, they frequently have difficulty in mastering the interrelationships and cross generalizations.' These broad generalizations apply to many dyslexic and dyspraxic primary school children, too. It is often observed that primary school dyslexic and dyspraxic children can be particularly rigid and inflexible in their thinking and that they tend to see numbers and calculations in *linear*, action based, and rather tunnel-like ways.

As we will see, inflexibility in working with numbers at a primary school level usually springs from what is increasingly called poor *number sense* and a related weak basic *number concept*. In essence this means that children view numbers and calculations in primitive ones-based ways. Fuson, Wearne, Hiebert et al. (1997) call a ones-based number concept a *unitary* concept of numbers. An over-reliance on counting in *ones* and an inability to see patterns and connections means that it is hard for children to develop the broader, flexible understandings in number-work which underpin the ability to make links with other aspects of number-work. As Ashcraft et al. (1996) and Gray (1997) suggest, this is in large part because children who are weak at maths use calculation methods (mainly counting) which place very big demands on working memory. Instead of developing increasingly complex webs of understanding, many dyslexic and dyspraxic children tend to think along isolated calculation tracks. An impoverished understanding of numbers and number relationships (which underpins a poor generalizing ability) has enormous repercussions, the most important of which have been well documented:

1. A large number of number of dyslexic and dyspraxic children can complete calculations presented in familiar, easily recognizable, standard ways, but cannot cope with unfamiliar presentations, or challenging tasks.
2. Dyslexic and dyspraxic children often have difficulty understanding which operation is involved in mixed word-problem work.
3. In mental calculation work, dyslexic and dyspraxic children often fail to select an appropriate 'figuring out' fact derived strategy or mental calculation method. In many instances, some dyslexic and dyspraxic children 'may be so confused us to have no clues as to where to start' (Chinn and Ashcroft, 1998, p. 8).

Diverging strengths and weaknesses

An interesting dyslexic maths learning personality

Although a large proportion of dyslexics have poor fact recall it has long been noted – by, for example, Tim Miles, Chinn and Ashcroft, and Anne Henderson – that a significant number of dyslexic children are good at the 'thinking', conceptual, or problem-solving aspects of mathematics. Teaching experience confirms that some dyslexic children solve certain maths tasks spectacularly quickly and without appearing to do much calculation. While fact-retrieval difficulties can slow down their calculation some dyslexic children quickly grasp the principles of logico-mathematical (or 'thinking') calculation strategies. With the right support, some dyslexic children are able to invent ways of figuring out difficult calculations for themselves. On the other hand, it is also commonly noted that the innate mathematical 'flair' of these dyslexic children can be difficult for them to harness, particularly in more traditional maths classrooms. Typically, dyslexic children who are able 'thinkers' are not able to explain the methods that they used to solve calculations or word problems. Since they appear to be working intuitively, they often seem genuinely unable to record their methods in written 'workings' or recording. In part because their exact fact knowledge is limited, their answers are often inaccurate, too.

A consideration of the numeracy profiles of dyspraxic children

Unfortunately there is very little available detail about the 'typical' numeracy profiles of dyspraxic children and general comments and teachers' responses tend to be somewhat gloomy. As we have already seen, Portwood notes that the majority of dyspraxic children have significant difficulties learning maths. Studies of children with 'spatial weaknesses' indicate that poor spatial ability correlates with weaknesses in the very earliest stages of number-work. One interpretation of this is that very early number-work depends on physical counting activities, and objects are often hard for children with spatial difficulties to manipulate; concrete work is also hard for children with poor spatial ability to process and visualize (Carter, Crawley and Lewis, 1999a). As we have already noted, too, teachers consistently report that the majority of dyspraxic children struggle to understand concepts, logico-mathematical ways of reasoning (such as fact-derived strategies, or mental calculation methods), word problems and number puzzles. It is often commented on that dyspraxic children seem to be particularly rigid maths thinkers. While there are no references in the available literature on dyspraxia or in 'mainstream' maths learning literature to surprising areas of *ability* among dyspraxic maths learners, some dyspraxic children are able to do reasonably well in certain areas of maths. As we have suggested, this is largely because some dyspraxic children have strong verbal memory abilities. While it is not the case that all dyspraxic children master *times tables* as verbal associations, it would seem that a small but significant proportion of dyspraxic children are able to do so. We have already commented on the fact that some dyspraxic children are able to learn the routine, procedural aspects of maths, and are able to perform well in familiar calculation situations.

Ways of interpreting the very different maths learning personality possibilities among dyslexic and dyspraxic maths learners

Very crudely speaking, able dyslexic maths learners and (relatively) able dyspraxic maths learners seem to have abilities at opposite ends of the broad spectrum of numeracy requirements. In simple terms, a small number of dyslexic children have 'thinking' abilities in maths whereas most dyspraxic children find the 'thinking' aspects of maths especially hard to manage. On the other hand, a small number of dyspraxic children have verbal memory abilities in maths whereas the vast majority of dyslexic children cannot remember verbally encoded facts and procedures.

There are two bodies of research and theoretical insights from within separate research paradigms which help cast some light on these opposing areas of strength. The first body of work, made very familiar by Steve Chinn, and explored by Chinn with different dyslexic maths learning *styles* in mind, is work which investigates the two very different maths personality types – the *grasshoppers*, at one end of the learning continuum, and the *inchworms*, at the other end of the continuum. The second body of work draws on the research into the 'maths areas' of the brain undertaken by the neuroscientists Brian Butterworth and Stanislas Dehaene. From the outset it should be made clear that neither Butterworth nor Dehaene have as yet directly studied the maths processing areas of the brains of dyslexic and dyspraxic individuals. However, Butterworth's and Dehaene's broader discoveries about the specialist maths processing regions of the brain, which they are busy mapping out, appear to offer some help with interpreting the dyslexic *grasshopper* strengths, on the one hand, as well as the rather extreme 'thinking' weaknesses of severely dyspraxic children and the very extreme verbal memory weaknesses of dyslexic children, on the other hand.

Maths learning personalities: different learning styles in mathematics

Although the American-based mathematician and maths learning specialist, Mahesh Sharma, has pursued broadly parallel investigations of maths learning personalities in the US, the work of Chinn and Ashcroft builds mainly on the research of Bath and Knox in the UK into how secondary school dyslexic children actually 'do' maths. To represent the two very different cognitive styles or maths learning extremes at opposite ends of a maths learning continuum Sharma uses the terms *qualitative* and *quantitative* maths learning personalities. Chinn and Ashcroft prefer Bath and Knox's suggestive terms, *grasshopper* and *inchworm*, to characterize the cognitive styles of dyslexic maths learners. Chinn and Ashcroft suggest that the learner's cognitive style is informed by his or her overall learning style.

Over time, many adults who are involved in thinking about dyslexia have found the *grasshopper–inchworm* distinction helpful. In particular, the description of *grasshopper* characteristics has helped parents and teachers better understand the learning needs – basically the need for informed and flexible, reasoning based teaching – of *grasshopper* children. Although

Bath and Knox, and Chinn and Ashcroft are careful to point out that many dyslexic children display both *grasshopper* and *inchworm* characteristics and that there are children at both 'extremes of the continuum' (Chinn and Ashcroft, 1998, p. 23) there has been a popular tendency to exaggerate somewhat the numbers of *grasshopper* dyslexic children. Involvement with some hundreds of dyslexic primary school children over the years has confirmed Chinn and Ashcroft's view that, among dyslexic children, 'there are more inchworms than grasshoppers' (Chinn and Ashcroft, 1998, p. 23).

More detailed accounts of 'grasshoppers' and 'inchworms' can be found in Chinn and Ashcroft (1992; 1998) and in Chinn et al. (2001). It is important, however, to give an overview description of the characteristics of *grasshoppers* and *inchworms*. The research of Bath and Knox, Chinn and Ashcroft, and Chinn, shows that dyslexics at the *grasshopper* end of the continuum approach maths in a 'holistic', global, conceptual, and intuitive way. *Grasshopper* dyslexics start from an appraisal – often a mental image, or picture – of the whole. In other words, *grasshopper* dyslexics usually solve tasks in what might be described as a top-down way. *Grasshoppers* are often the inventive and creative thinkers and good problem solvers whom we described a moment ago. However, research also shows that dyslexic *grasshoppers* have poor knowledge of maths facts and often pay poor attention to detail. Although some *grasshopper* children are able to use their thinking skills to circumvent their maths facts difficulties, dyslexic *grasshoppers* can be vulnerable in maths, as Chinn and Ashcroft stress, because their intuitive understanding can be undermined by poor facts knowledge and working memory difficulties. Chinn et al.'s (2001) three-year study of adolescent *grasshopper* and *inchworm* dyslexics in three different countries confirms that secondary school *grasshoppers* do particularly well in 'open' and reasoning-based learning environments, such as Realistic Education classrooms in The Netherlands, and Mark College.

Dyslexic children with *inchworm* styles solve tasks in a piecemeal or bit by bit way. *Inchworm* dyslexics cannot overview or visualize maths tasks. Instead, they approach tasks from what they believe is the first step and proceed in a linear step-by-step way until they reach the end of the task. Because *inchworms* work in a laborious 'bottom-up' way and cannot picture problems they have no sense of what the outcomes of maths tasks are likely to be. As Sharma (1989a) points out, traditional maths teaching approaches reinforce *quantitative* or *inchworm* maths learning styles. Although literature searches have not yielded any research into the 'maths learning personalities' of dyspraxic children, teaching experience bears out that dyspraxic maths learners have much in common with descriptions of dyslexics whose learning styles are situated towards the *inchworm* extreme of the maths personality continuum.

Maths 'in the brain': a neurophysical paradigm

As we have seen, the study of the learning behaviour of maths pupils and of the behaviour of dyslexic pupils as a special category of maths learner has led to the helpful idea that maths personalities (or learning styles) can be analysed and categorized along a continuum between maths personality extremes. In very general terms it may be said that the starting point for the new and very exciting work of those neuroscientists who are interested in exploring maths 'in the brain' is to ask the underlying and most fundamental *why*? questions. Neuroscientists set out to go behind observable maths behaviours to try and understand the brain-related aspects of maths considered as a special knowledge domain.

In very recent years, the neuroscientists Brian Butterworth (1999) and Stanislas Dehaene et al. (1999) have used various research techniques to begin the process of locating the parts of the brain which are used in thinking about numbers and in calculation. The research methods include testing the responses of stroke patients to various number-related maths tasks and using MRI scanning or functional magnetic resonance imaging. Although Brian Butterworth and Stanislas Dehaene have slightly different ideas about where and how numbers are represented in the brain their research also has important features in common. First of all, the new neuroscientific research seems to show that an understanding of numbers, and most processing involving numbers, takes place in a special brain site which is a non-language area of the brain. Neuroscience seems to show that an understanding of numbers is not, in the first instance, a 'special aspect of language' and does not derive in the first instance (as Piaget believed) from 'more primitive logical concepts,' either (Butterworth, 1999, p. 166). In other words, many aspects of number processing are specialized and are language independent, although both language and logic are thought to be implicated in the process of learning some of the more complex aspects of any culture's accumulated maths knowledge. Secondly, and broadly speaking, Butterworth's and Dehaene's work confirms that the key area for understanding and working with numbers is situated in the *parietal lobes* of the brain – a non-language area of the brain. Overall, the research suggests that the parietal lobes of the brain are activated or involved in most numerical processes, but Dehaene et al.'s (1999) brain imaging research also points to the particular importance of the parietal lobes for intuitive, visuo-spatial, and non-verbal ways of representing numbers. It should be noted that the broader functions of the parietal lobes include visuo-spatial processing, visually guided hand–eye coordination, finger control and attention orientation (Dehaene et al., 1999). Thirdly, Butterworth and Dehaene both believe that we have very specialist brain circuits, which are inborn, or are at least present very early, which represent an 'inner core' of number ability, and which underlie all subsequent numerical development. Brian Butterworth (1999) calls this core number area an 'inner core start-up kit' and, more specifically, 'The Number Module'. Butterworth's research leads him to believe that this core number area is in the left parietal lobe of the brain.

From a teaching point of view, Dehaene's further studies of where and how a very distinctive aspect of *institutionalized* arithmetical knowledge – namely, immediately known maths facts – are represented, is also of particular value. Dehaene et al. (1999) brain-imaging research shows that the knowledge of immediately recalled 'exact facts' seem to involve a 'language dependent mode' of representation. According to Dehaene, the site in the brain of the language dependent number mode is in the left hemisphere of the brain, in the left inferior frontal lobe. The particular significance of this location is that the left inferior lobe is a part of the brain which is known to be responsible for verbal associations, such as generating verbs for given nouns. Dehaene has also measured the response times of bilingual students to maths facts questions: the response times seemed to demonstrate that exact facts are generally stored by the brain as 'verbal associations', and probably as 'exact sequences of words'. According to Dehaene, the responses of students seem to show that 'exact fact arithmetic' is typically represented in a language specific format and, that where a language format is used, this 'transfers poorly to a different language or to novel facts, and recruits networks involved in word association processes' (Dehaene et al., 1999, p. 970).

An overview of the maths personality and neuroscientific paradigms: some speculative thoughts

Dyslexics and maths facts

As we have seen, it is a characteristic feature of dyslexics that they have difficulty recalling exact maths facts. In general terms, dyslexic children appear to demonstrate broad left-hemispheric weaknesses. It seems possible that many dyslexics may have weaknesses of the left inferior frontal lobe. As we will see in Part V, it is striking that it is precisely the 'exact sequences of words' or 'verbal associations' that so many dyslexics fail to remember. Miles in Miles and Miles (1992, p. 2) referred to the dyslexic pupils' difficulty with maths facts as a difficulty with 'paired associate learning'.

Grasshopper dyslexics

Thus far, we have concentrated in the main on those dyslexic children who have maths learning difficulties. However, as we have also suggested, a proportion of dyslexic children (estimates vary between quite a small percentage, to about 50 per cent, depending on how 'difficulty' is defined) do not have any significant degree of difficulty learning maths. The neuroscientific finding that numbers are largely processed in a non-language area is clearly significant in this regard. In other words, dyslexic children may have 'impairments' in language areas only or – and this will mean that the child is likely to have more 'global' maths learning difficulties – they may have impairments in language areas *and* in number areas of the brain. As we have just seen, research and teaching experience and informal accounts indicate that some dyslexics who have poor maths facts knowledge nevertheless also have intuitive and seemingly visuo-spatial maths insights. Chinn often suggests that *grasshopper* dyslexics may become more *grasshopper-like* to compensate for poor 'in one' maths facts knowledge. As we have seen, dyslexia is generally associated with relative right-hemispheric strengths. From the general implications of 'maths-in-the-brain' studies it seems possible that *grasshoppers* may have relative parietal lobe strengths. In other words, brain studies seem to suggest that dyslexics are able to be 'gifted problem solvers' if they have 'spatial mode' or visuo-spatial strengths in the parietal lobes

Dyspraxia

Definitions of dyspraxia include visuo-spatial weaknesses as a characteristic feature of dyspraxia. We have seen that dyspraxia is associated with general *performance* weaknesses. The insights from brain studies into the different modes of representing numbers is highly suggestive for understanding many of the number difficulties of moderately to severely dyspraxic children. In fact, the list of broader functions of the parietal lobes, or site of visuo-spatial representation of numbers, reads like a list of many of the 'typical' dyspraxic features. This suggests that many dyspraxics may have a degree of parietal lobe impairment. This would help explain why dyspraxics have 'poor intuitions' about maths tasks, and seem unable to visualize unfamiliar problems, challenging tasks, or maths puzzles. It would also help explain why dyspraxic children have difficulty understanding and remembering logico-mathematical ways of thinking in number-work. The inability to picture tasks would clearly underpin the tendency for dyspraxic children to display inflexible and 'bottom-up' *inchworm* maths processing characteristics.

Dyspraxia, and verbal strengths

We have seen that dyspraxia is associated with relative left-hemispheric strengths. Although most dyspraxic children are poor thinkers and problem solvers in maths and many dyspraxics share the verbal memory weaknesses of dyslexic children, a few dyspraxics have good verbal maths skills. This suggests that some dyspraxic children may prefer to represent numbers and maths facts whenever possible in the 'language dependent mode' and may have relative left inferior frontal lobe strengths. Such strengths, combined with visuo-spatial weaknesses, would probably confirm *inchworm*-like tendencies, even among relatively able dyspraxic maths learners.

Some teaching implications

In general terms it is important to bear in mind that dyslexia and dyspraxia are often comorbid and that dyslexic and dyspraxic children with difficulties in maths have many cognitive characteristics in common, as we saw earlier on. This means that the fundamental teaching principles for teaching maths to dyslexic and dyspraxic children are broadly the same. These principles are outlined in detail in Chapter 2. Nevertheless the considerations of the different cognitive, biological and learning personality strengths and weaknesses which can impact on individual children with specific learning difficulties do have some important teaching implications.

In brief, the strong visuo-spatial skills of *grasshopper* dyslexics lend themselves to visuo-spatial modes of representing numbers and number relationships. Mahesh Sharma (1989b) points out that *qualitative thinkers (grasshoppers)* identify with 'continuous' or spatially defined materials, such as Cuisinaire rods (these and other maths materials are described later on). Most dyslexic *grasshoppers* quickly make sense of spatial models of maths operations such as 2-D 'area models' of multiplication, for instance. Since *grasshoppers* reason in a 'top-down', holistic way they find the column-based standard models of calculation particularly difficult to make sense of. On the other hand, as we have noted, most dyslexic *grasshoppers* thrive on mental methods of calculating and, if encouraged, will often devise sophisticated methods of calculating for themselves. Since they work intuitively, most dyslexic *grasshoppers* have to be encouraged to record reasoning steps or partial calculations. Mental calculation methods allow *grasshopper* dyslexics to develop methods for recording their 'workings' which will support their ways of visualizing the tasks

On the other hand, the poor visualizing skills of severely dyspraxic children and dyslexic *inchworms* means that they can find spatially defined models of numbers and spatially defined materials – such as Cuisinaire rods – quite difficult to make sense of. As Sharma points out, *quantitative thinkers (inchworms)* seem to prefer discontinuous or discrete models and materials – such as small dots, or counters and cubes. Since most dyspraxic children and most dyslexic inchworms benefit from being helped to develop strongly defined and economical visual images for number relationships, it is generally important to encourage them to build and use visuo-spatial models. To ensure that dyspraxic children and dyslexic *inchworms* genuinely understand *qualitative* models and pictures it is often wise to use ones-based (discrete) models in order to introduce new concepts and relationships to them. When the ones-based models are understood, teachers can build up towards an understanding of spatially defined models. (It should be noted, however, that some severely dyspraxic children seem unable to make sense of predominantly spatial ways of representing relationships, even when they are carefully introduced.) The poor visualizing skills of dyspraxic children and dyslexic *inchworms* also affects their ability to understand certain logico-mathematical ways of reasoning. For example, children with poor visuo-spatial skills find it very difficult to understand 'holistic' *compensation* methods of reasoning, such as $38 - 19 = (38 - 20) + 1$. Finally, teaching experience has shown that the extremely poor visuo-spatial skills of very severely dyspraxic maths learners can seriously hamper the ability of these children to subtract or divide: it would appear that children with severe visuo-spatial weaknesses find it easier to visualize the processes of putting quantities or numbers together (addition and multiplication) than breaking quantities or numbers apart (subtraction and division).

A note on the term dyscalculia

The label dyscalculia is quite often used to describe an inherent and severe difficulty with acquiring numerical skills. From the start, the idea that there may be a specific mathematics difficulty has been controversial. Some respected figures in the dyslexia world, for example Tim Miles, believe that dyscalculia is really part of other specific learning difficulties, such as dyslexia, and that there is no need for yet another label.

In very general terms, there is a broad consensus among those experts who favour using the term dyscalculia about how it should be characterized:

1. Dyscalculia is described as a global arithmetic learning difficulty. It is understood to affect all aspects of basic numeracy. Dyscalculic learners have a poor intuitive sense or *feel* for numbers and for number relationships. They usually have difficulty learning facts and procedures. They have problems understanding concepts and logical principles. They have grave difficulties understanding the number system. Some dyscalculics are able to acquire facts and procedures but give answers or solve tasks mechanically and with little understanding.
2. Dyscalculia is described as a very severe number learning difficulty. The mathematical attainment levels of dyscalculic children are very significantly lower than their peers: at age 7, for example, dyscalculic children could be functioning at the level of an average 4–5 year old. By 11 years of age dyscalculic children are often 5 or 6 years behind their peers in number-work. Some dyscalculic adults are still functioning at the level of a young primary school child. It is often noted that the majority of dyscalculic children (and even adults) complete calculations by counting in *ones* and that they often use their fingers to do so.
3. The label dyscalculia is not usually used to apply to individuals with severe *general* intellectual impairment. It is generally reserved for individuals who otherwise function fairly normally. The term dyscalculia is understood to be meaningful when there is a discrepancy between overall intellectual ability and levels of attainment and the individual's arithmetical ability.

Many experts who value the term dyscalculia nevertheless disagree about whether it should be seen as an entirely separate learning difficulty (a specific and separate difficulty with numbers, only) or whether it should be understood as comorbid with, and sharing common underlying causes with, other specific learning difficulties. Although the arguments about whether dyscalculia is linked to, or entirely separate from, *language processing difficulties* are too complex to explore here, it is possible that some of the debates about dyscalculia have been limited by omitting considerations of dyspraxia from the debates; instead, many discussions focus on a *dyslexia-versus-dyscalculia* frame of reference, alone. Although the theoretical debates about dyscalculia will probably only begin to be resolved when we have a better understanding of 'the mathematical brain' – when scientists have begun to map out in detailed ways which parts of the brain are affected in a wide range of children and adults of different ages who are 'very poor at numbers' – it is also possible to make some comments from a pragmatic teaching point of view:

1. As Chinn and Ashcroft (1998, p. 3) argue, the percentage of children who have 'learning difficulties which are solely related to mathematics' seems to be small. Most children who experience severe difficulties in learning about numbers also seem to have another attributable specific learning difficulty, such as dyslexia or dyspraxia. As Chinn and Ashcroft suggest, children with number difficulties and with no other learning difficulty do exist and it is obviously valid to label such children dyscalculic but it seems unduly limiting to circumscribe the label 'dyscalculia' to such children, alone.
2. As we have seen, a significant proportion of children with language difficulties (dyslexic children) also have severe difficulties learning maths.
3. Dyspraxia seems to be highly correlated with severe and deep-seated maths learning difficulties. It should also be noted that many research definitions of the cognitive features of children with *maths learning difficulties* but with no significant language learning difficulties (for example, in Sharma, 1986) correlate very strongly with descriptions of dyspraxia. In particular, visuo-spatial difficulties are found by many researchers to be a key underlying feature of children with so-called *specific maths learning difficulties* (Sharma, 1986).
4. Teaching experience has shown that the vast majority of children with very severe maths learning difficulties – who are functioning many years behind their peers – can be helped to make reasonable and sustained progress. Sharma (1986) suggests that a key to enabling dyscalculic children to make progress in maths is to help them break with the habit of counting in *ones*. This idea is a key theme informing the learning approach described in this book. In contrast to a 'counting

habit', the teaching proposals contained in this book set out to foster a 'reasoning habit' – although it is also important to acknowledge that counting can persist in certain calculation situations and that counting responses also typically re-emerge under stress. Teaching experience has also shown, however, that there are also a very few children who seem to have such a disastrously poor intuitive grasp of numbers and who are so inflexible in their thinking about numbers that they seem unable to make significant progress in learning to work with numbers, at all. It is tempting to speculate that a very important and specialized core number processing part of the brain – possibly Butterworth's 'start up kit' or Number Module – is very significantly impaired in such children. At present there seems to be no available knowledge about how to teach maths successfully to such extremely severely impaired children.