# INTERMARKET TRADING STRATEGIES

Markos Katsanos

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Intermarket Trading Strategies

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*To my wife Erifili*

# Contents

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Special thanks to my wife, Erifili, for her love and patient support, and for putting up with late nights, despite having no actual interest in the subject.

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It is difficult to appreciate the effort and hours that go into researching and writing a book of this nature until you've had a chance to work on one yourself. The correlation studies involved a huge amount of historical data and I would like to thank Peter Day for helping out with the VBA to manage the data in Excel and Rubi Cabal, marketing director at CSI data, for allowing me access to CSI's correlation reports.

Finally, I would like to thank the technical support staff at Ward Systems Inc., and also Lynn Dufrenne, Lead Support Representative at Equis Technical Support for their help in solving the unavoidable software problems.

# Introduction

With the emergence of the internet and international cross border trading, the world's futures and equity markets started to converge, making intermarket analysis an essential constituent of technical analysis.

Single market technical analysis indicators were designed in the 1980s for national markets, and are no longer sufficient nor can be relied upon for analyzing the constantly changing market dynamics.

But how do markets interact and influence each other and how can we use intermarket relationships to construct a viable technical system?

Because the answers have been so elusive, they became the motivation for my research. The more I looked into it, the more I became convinced that there is clear and incontrovertible evidence that the markets are linked to each other, and incorporating intermarket correlations into a trading discipline can give a trading advantage. Two years of research later, I finally came up with some rules and mathematical formulae for intermarket trading.

It has been more than 15 years since John Murphy, a pioneer on the subject, wrote his first book on intermarket analysis. The material in my book is based on original research not published anywhere else and, unlike Murphy's intuitive chart-based approach, I am going to use mathematical and statistical principles to develop and design intermarket trading systems appropriate for long- and short-term and even day trading.

Although the book makes extensive use of market statistics obtained from hundreds of correlation studies, the data and empirical findings are not its heart. They serve as a background in developing the trading systems presented in the second part of the book, as well as help shape our thinking about the way the financial markets work.

The key difference between *Intermarket Trading Strategies* and other books on trading lies in its philosophy. I believe that knowing how the markets work is, in the end, more important than relying on a "black box" mechanical system that produced profitable trades in the past but not even the creator of the system can fully explain why. The focus of this book is how intermarket analysis can be used to forecast future equity and index price movements by introducing custom indicators and intermarket-based systems. A total of 29 conventional and five neural network trading systems are provided to trade gold, the S&P ETF (SPY), S&P e-mini futures, DAX and FTSE futures, gold and oil stocks, commodities, sector and international ETF, and finally the yen and the euro.

Naturally, past results are no guarantee of future performance.

Even so, the results of out-of-sample back-testing are compelling enough to merit attention. Some results are more compelling. The multiple regression gold system, presented in Chapter 11, returned an amazing $1.2 million of profits on a $100 000 initial equity. The stock index trading systems also produced impressive profits. The profitability of the Standard & Poor's e-mini intraday system was neither standard nor poor, producing a 300 % profit during the test duration. Investors who prefer to trade only stocks will find intermarket systems for trading oil and gold stocks in Chapter 15. My favorite is the oil stock system which made more than $1.6 million on an initial equity of only $100 000.

The foreign exchange (or Forex) market, which until recently was dominated by large international banks, is gaining popularity among active traders, because of its superior liquidity and 24-hour trading. Readers who are interested in forex trading will find, in Chapter 17, an intermarket system for trading the yen which made over $70 000 on an initial $3000 account. The EUR/USD is the most popular currency pair among forex traders and the chapter on forex wouldn't be complete without a system for trading the euro. The next section in Chapter 17 presents two systems: a conventional and a hybrid system. The latter is an excellent example of how you can enhance a classic system by adding intermarket conditions. The hybrid system improved considerably on the profitability of the traditional trend-following system, almost doubling the profit factor while reducing drawdown.

The system design is fully described from the initial concept to optimization and actual implementation. All systems are back-tested using out-of-sample data and the performance statistics are provided for each one.

The MetaStock code for all systems is provided in Appendix A and a detailed procedure for recreating the artificial neural network systems in NeuroShell Trader is included in Appendix B.

The benefits of diversification, and an example of static portfolio diversification by optimizing the portfolio allocation based on the desired risk and return characteristics, are discussed in the first chapter and a dynamic portfolio allocation method, based on market timing and relative strength, is included in Chapter 16.

The book is divided into two parts. Part I serves as a background to Part II and includes an overview of the basics of intermarket analysis, correlation analysis in Chapter 2 and custom intermarket indicators in Chapter 9.

Part II uses many of the concepts presented in Part I to develop custom trading systems to trade popular markets like US and European stock index futures, forex and commodities.

The chapters in this book are divided as follows.

- The book begins with a discussion of the basic principles of intermarket analysis and the benefits of portfolio diversification by including uncorrelated assets such as commodities and foreign currencies.
- Chapter 2 explains the concept of correlation and the basic assumptions used.
- Chapter 3 explains the linear regression method used for predicting one security based on its correlation with related markets. An example using the S&P, the VIX and the Euro Stoxx is also included. Nonparametric regression (involving nonlinear relationships) is also briefly discussed.
- Chapter 4 is an overview of major international financial, commodity and equity indices including the DAX, the CAC 40, the FTSE, the Euro Stoxx 50, the Nikkei 225, the Hang Seng, the dollar index, the oil index, the CRB index, the Goldman Sachs Commodity Index, the XAU, the HUI, and the Volatility Index (VIX).
- Chapter 5 examines the correlation between the S&P 500 and major US and international equity, commodity and other indices.
- Chapter 6 examines long-term, short-term and even intraday correlations between major European indices and stock index futures and explains how they can be applied to develop trading systems.
- Chapter 7 is about gold and its correlation with other commodities, indices and foreign exchange rates with special emphasis on leading/lagging analysis.
- In Chapter 8 of this book I examine intraday correlations, especially useful for short-term traders. A leading/lagging correlation analysis between the S&P e-mini, the Dow e-mini, DAX and Euro Stoxx futures is also included.
- Chapter 9 presents various custom intermarket indicators and explains how each one can be used within the framework of a trading system. These include eight new custom intermarket indicators, published for the first time in this book. The MetaStock code for all indicators is included in Appendix A.
- The personal computer has revolutionized the creation and testing of trading systems. The inexpensive software takes out much of the work needed for testing, allowing the trader to test endless permutations of rules and parameters. In Chapter 10, I present some techniques for developing a trading system and evaluating the test results. I suggest methods of avoiding curve fitting and the illusion of excellence created by optimization. I also discuss stop-loss and other money management techniques. Finally, a brief introduction to neural network systems will explain the basic principles of this alternative approach for designing trading systems.
- Chapter 11 compares the performance of 15 intermarket systems for trading gold based on the intermarket indicators in Chapter 9 and using gold's correlations with the dollar index, the XAU and silver.
- Chapter 12 describes a long-term market timing system for trading the S&P ETF (SPY) using a market breadth indicator, and for traders who prefer the emotionally charged world of short-term trading, a hybrid system for trading the S&P e-mini using 5-minute intraday correlation with Euro Stoxx 50 futures.

- Chapter 13 presents intermarket systems for trading DAX futures based on its correlation with Euro Stoxx and S&P 500 futures.
- Artificial neural network systems are gaining popularity but are they the panacea that software vendors claim them to be? A comparison between a conventional rule-based system and a neural network strategy for trading the FTSE in Chapter 14 will shed some light on this innovative trading method, exposing the advantages and disadvantages of using neural network-based systems in predicting future price movements using intermarket relationships.
- Oil and gold have been so far (through June 2008) two of the best performing commodities in 2007–2008. For the benefit of readers who prefer to trade only stocks, Chapter 15 presents examples of intermarket trading systems for trading oil and gold stocks, using their correlation with the corresponding sector indices.
- Chapter 16 presents a dynamic asset allocation strategy by trading across different asset classes according to their relative strength.
- Forex trading is also gaining popularity and the final chapter (which is, by the way, the longest chapter of the book) is all about forex. Chapter 17 is divided into four parts. The first part discusses fundamental and technical factors affecting the foreign exchange markets and intermarket correlations between currencies, commodities, interest rates and equities, with a brief explanation of the carry trade. The correlation analysis included in the first part is used to develop an intermarket system to trade the yen and a hybrid system for trading the euro. The Australian dollar has received considerable attention lately as it is the favorite currency of "yen carry" traders. The last part of this chapter contains fundamental factors affecting the Australian dollar and its correlation with major international indices, commodities and other currencies.

The systems presented in this book are by no means exhaustive, and they are limited by space and the author's imagination and experience. They are presented as a challenge to the serious trader to reevaluate intermarket techniques as a working tool and to introduce a testing framework which amalgamates both intermarket and classic technical analysis indicators.

This book does not require any mathematical skills beyond those taught in high school algebra. Statistical and correlation analysis is used throughout the book but *all* of the theoretical tools are introduced and explained within. All other concepts (such as indicators, trading system evaluation, neural networks, etc.) are motivated, defined, and explained as they appear in the book.

Although all systems (except neural networks) are written in the MetaStock formula language it is not necessary to purchase MetaStock in order to use them, as they are fully explained in English and the MetaStock functions used are explained in the Glossary.

Not long ago, I ran into an acquaintance who asked me why I need to write a book about trading systems if I can make more money by trading in a day than by the royalties from the book in a year. I don't think that it makes a lot of sense to write a book for money and I don't know any author who became rich from writing (unless the book is a huge success). There are, of course, other valid reasons that motivate people to write books, like impressing your friends or establishing yourself as an expert in the subject or even visibility and media coverage. Although I have to admit that all these have crossed my mind, my main reason was the motivation to research the highly complex subject of intermarket relationships and write down my findings in an orderly, methodical and timely manner (which I wouldn't have done had I not had the deadlines imposed by my contract with the publisher).

I hope that readers will find the information useful and exploit my intermarket indicators and strategies in order to detect profitable trading opportunities.

# Part I

# 1
# Intermarket Analysis

*It's not that I am so smart; it's just that I stay with the problems longer.*

–Albert Einstein

The basic premise of intermarket analysis is that there is both a cause and effect to the movement of money from one area to another. Consider, for example, the price of gold and the dollar. Because gold is denominated in US dollars, any significant fluctuation of the dollar will have an impact on the price of gold, which in turn will affect the price of gold mining stocks.

The strength and direction of the relationship between two markets is measured by the correlation coefficient which reflects the simultaneous change in value of a pair of numeric series over time.

Highly positively correlated markets can be expected to move in similar ways and highly negatively correlated markets are likely to move in opposite directions. Knowing which markets are positively or negatively correlated with a given market is very important for gaining an understanding of the future directional movement of the market you propose to trade.

Advancements in telecommunications have contributed to the integration of international markets. Sophisticated traders are starting to incorporate intermarket analysis in their trading decisions through a variety of means ranging from simple chart analysis to correlation analysis. Yet the intermarket relationships hidden in this data are often quite complex and not readily apparent, while the scope of analysis is virtually unlimited.

But what is intermarket analysis?

The financial markets comprise of more than 500 000 securities, derivatives, currencies, bonds, and other financial instruments – the size of a small city. All interact with each other to some extent and a seemingly unimportant event can cause a chain of reactions causing a landslide of large-scale changes to the financial markets.

Consider the following example: Let's suppose that the Bank of Japan decides to buy dollars in order to push the yen down. As a result Japanese stock prices will go up as a weak yen will help boost profits for exporters. A sharp rise of the Nikkei will in turn have a positive effect on all other Asian markets. The next morning European markets, in view of higher Asian markets and in the absence of other overnight news, will open higher. This will in turn drive US index futures higher and boost US markets at open. In addition lower yen prices will encourage the "yen carry trade", i.e. borrowing yen at lower or near zero interest rates and buying higher yielding assets such as US bonds or even emerging market equities, which in turn will push bonds and equities higher. On the other hand, a scenario for disaster will develop if the opposite happens and the yen rises sharply against the dollar. This will cause a sharp unwinding of the "yen carry trade", triggering an avalanche of sharp declines in all financial markets.

But what might cause the yen to rise? The following is a possible scenario: As we head into the economic slowdown, the carry trade money that has flowed into risky cyclical assets is likely to fall in value. As a result, speculators in these assets will cut their losses, bail out and repay their yen debts. This is a scenario for disaster because when the yen rebounds against the dollar, it often snaps back very fast and carry trades can go from profit to loss with almost no warning.

A popular chaos theory axiom (known as the "butterfly effect" because of the title of a paper given by the mathematician Edward Lorenz in 1972 to the American Association for the Advancement of Science in Washington, D.C. entitled "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?") stipulates that a small change in the initial condition of the system (the flapping of the wing) causes a chain of events leading to large-scale phenomena. Had the butterfly not flapped its wings, the trajectory of the system might have been vastly different.

A financial series would appear to be chaotic in nature, but its statistics are not because, as well as being orderly in the sense of being deterministic, chaotic systems usually have well defined statistics.

The rapid progress of global communications has contributed to the integration of all international financial markets as the world has gotten smaller due to the ability to communicate almost instantaneously. Relationships that were dismissed as irrelevant in the past cannot be ignored any more as the globalization of the markets contributes to a convergence of formerly unrelated markets.

Take a look at the comparison chart in Fig. 1.1. The S&P 500 is depicted with a bold thick line. The second one however is not even a stock index. It is the Japanese yen exchange rate (USD/JPY).

The next composite chart in Fig. 1.2 is of three stock indices. The first two (depicted with a bar chart and thick line) are of the S&P 500 and the Nasdaq Composite respectively. The third chart (thin line) is the Athens General Index which, surprisingly, correlates better with the S&P 500 than its compatriot, the Nasdaq Composite.

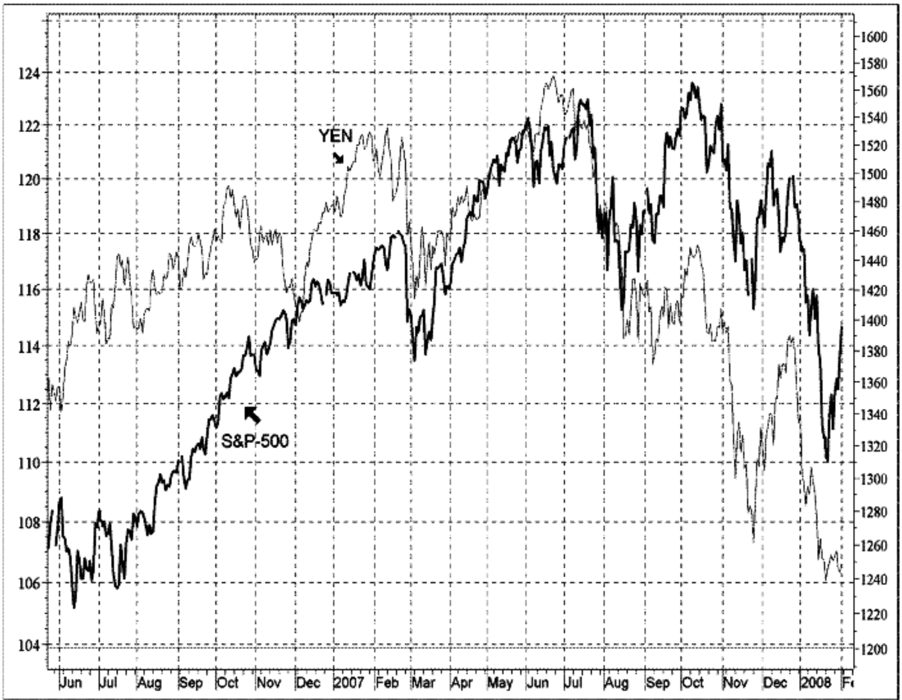


**Figure 1.1** Comparison chart of the S&P 500 (in bold with the scale on the right Y-axis) and the yen (USD/JPY) (with the scale on the left axis) from June 2006 to January 2008.

The above examples are included to illustrate that the integration of global markets can extend beyond the obvious relations.

I often hear CNBC guests suggesting investing in international markets as a means of diversifying one's portfolio away from the US equity markets. Although some emerging markets may have relatively medium to low correlation with US markets, one important question to ask is whether diversification works when it is needed most. Evidence from stock market history suggests that periods of negative shocks and poor market performance were associated with high, rather than low, correlations. The events of 21 January 2008 are still fresh in my mind, when a 2.9 % correction in the S&P 500 was followed the next day by a devastating 7.2 % drop in the German DAX, wiping out nine months of profits in a day. Emerging markets sunk even more with the Jakarta Composite falling more than 12 % in two days while Brazil's Bovespa lost more than 8.5 %. Indeed, investors who have apparently relied upon diversification in the past to protect them against corrections of the market have been frequently disappointed.


**Figure 1.2** Weekly comparison chart of the S&P 500 (thick line with the scale on the right Y-axis), the Nasdaq (in bold with the scale on the left axis) and the Greek Athens General Index (ATG) from 1999 to 2008.

The only effective method of diversifying one's portfolio is by including asset classes with low or negative correlation to stocks such as cash, foreign exchange or commodities. Whatever the relationship is – leading, lagging, or divergent responses to economic conditions – a strong negative correlation coefficient between two markets is a suggestion that these markets will move against each other sometime in the future. And, of course, the higher the absolute value of the coefficient of correlation, the higher the diversity of their performances.

Although intermarket analysis has been classified as a branch of technical analysis, it has not been embraced fully by analysts. The majority of traders continue to focus on only one market at a time and they tend to miss the forest for the trees. No market exists in a vacuum, and traders who focus on the bigger picture portrayed through all international markets tend to be the ones that deliver better performance.

Traditional technical analysis indicators such as moving averages are lagging indicators calculated from past data and are limited in assessing the current trend. Regardless of the hours spent in back-testing, there is a limit beyond which a system based on a lagging indicator can be improved further. Thus the addition of leading indicators that anticipate reversals in trend direction is essential and beneficial to the system's performance. These can only be created by taking into consideration directional movements of correlated markets.

The use of intermarket correlation analysis can help you improve on your trading system by avoiding trades against the prevailing direction of correlated markets, but can also be used on its own to develop a complete system based on divergences between two or more highly correlated markets. Knowing the correlation of the market you propose to trade with other markets is very important for predicting its future direction. In addition, short-term traders can take advantage of the time difference between world markets and anticipate the next day's movement. Asian markets are the first to start trading, followed by the European markets. For a US trader the insight gained from all preceding markets is a valuable tool in predicting at least the opening in his local market.

I have found that the most accurate economist is the market itself. It is far easier to forecast economic activity from the behavior of markets themselves than it is to forecast the capital markets from lagging economic statistics such as the unemployment index. The market is a discounting mechanism. It interprets the impact of economic news some time in the future. Of course, this is only a guess and guesses are not always right. But the truth is that the market is a much better guesser than any of us are, as it represents the average opinion of all the economists in the world.

There appears to be no end to the conclusions that can be drawn if a little understanding, imagination, and pure common sense are applied. Major changes in commodity prices affect the bond markets of different countries in different ways, depending upon their economic structure.

What sectors are affected first? Which asset class will provide the best potential profits? If opportunities dry up in one sector, where is the money heading to take advantage of the next cycle? This is what intermarket analysis can tell you if you learn what to look for, which makes it a grand endeavor and a continuing challenge but always worth the effort.

Intermarket analysis can also be useful in estimating the duration and state of the business cycle by watching the historic relationship between bonds, stocks and commodities as economic slowing favors bonds over stocks and commodities.

Near the end of an economic expansion bonds usually turn down before stocks and commodities and the reverse is true during an economic expansion. Bonds are usually the first to peak and the first to bottom and can therefore provide ample warning of the start or the end of a recession. Bonds have an impressive record as a leading indicator for the stock market, although this information cannot be used in constructing a trading system as the lead times can be quite long, ranging from one to two years.

You can see in Fig. 1.3 that bonds peaked in October 1998, 18 months before stocks peaked in March 2000 and 29 months before the official start of the recession in March 2001. The Commodity Research Bureau (CRB) index was the last to peak, making a complex triple top formation with the last peak coinciding with the start of the recession.

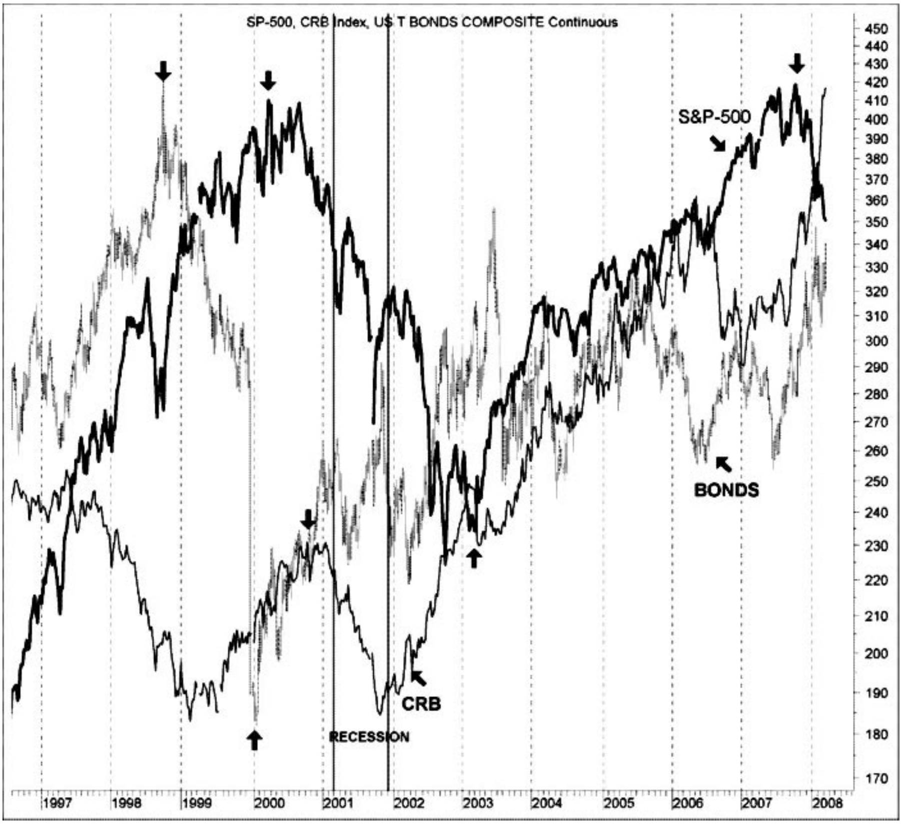


**Figure 1.3** Weekly composite chart of the S&P 500 (thick line), the CRB index (thin line with the scale on the right Y- axis) and US Treasury Bonds (grey bar chart) from 1997 to 2008. Down arrows indicate major tops in stocks and bonds and up arrows bottoms.

Bonds were also the first to bottom in anticipation of the recovery, followed by commodities and then stocks. From the beginning of 2003 until the middle of 2005 all three were rising together. Commodities are usually the last to bottom during a recovery but this was not the case here as they were boosted by the weakness in the dollar. The dollar made a final peak in January 2002 and reversed direction, dropping like a rock against the euro and other major currencies. This triggered a secular bull market in gold which spread to the rest of the commodities and has continued until the end of June 2008, almost nine months after stocks peaked in September 2007.

More information on the business cycle, including sector rotation during economic cycles, can be found in John Murphy's excellent book *Intermarket Analysis: Profiting from Global Market Relationships* (see Bibliography).

## 1.1 DETERMINING INTERMARKET RELATIONS

The simplest and easiest method of intermarket analysis is a visual inspection of a comparison chart of one security superimposed on the chart of another. A custom indicator can also be calculated from the ratio of prices, to help assess their past relation and anticipate future direction. Both of the above methods, however, are limited to two markets and the use of the correlation coefficient is essential for an analysis of multiple markets. For predictive purposes, we wish to detect correlations that are significantly different from zero. Such relationships can then be used to predict the future course of events in trading systems or forecasting models.

In addition, linear regression can be used to predict the future price trend of a market based on its correlation with multiple related markets.

When assessing intermarket relations you should always keep in mind that these are neither fixed nor static in time. Instead they fluctuate continuously in strength and time. It is usually very difficult to determine which market is leading or lagging. A lead can shift over time and become a lag, with the markets switching positions as follower and leader. In addition, a weak positive correlation can sometimes become negative and vice versa. For this reason it is always prudent to look at the prevailing rate of change of the correlation between two related markets before reaching any important conclusions or trading decisions.

The variability of the correlation over time is more evident in Fig. 1.4, where yearly correlations between the S&P 500 and four major international indices are plotted against time from 1992 up to the end of 2007. You can see that correlations before 1996 were inconsistent and unpredictable but started to converge during the last ten-year period. The most incongruous relationship is that between the S&P 500 and Japan's Nikkei (in white) as it fluctuated from negative to positive values over time.

The recent integration of global markets has also been accelerated by a flurry of mergers, acquisitions and alliances between international exchanges, the most important being the merger between the New York Stock Exchange and Euronext, Europe's leading cross-border exchange, which includes French, Belgian, Dutch and Portuguese national markets. A few months later the Nasdaq, after a failed bid for the London Stock Exchange, announced a takeover of the OMX, which owns and operates stock exchanges in Stockholm, Helsinki, Copenhagen, Reykjavik (Iceland) and the Baltic states.

## 1.2 USING INTERMARKET CORRELATIONS FOR PORTFOLIO DIVERSIFICATION

The benefits of diversification are well known: most investment managers diversify by including international equities, bonds and cash in their US stock portfolio. Less common, however, is the diversification into other asset classes such as commodities or foreign currencies (forex).

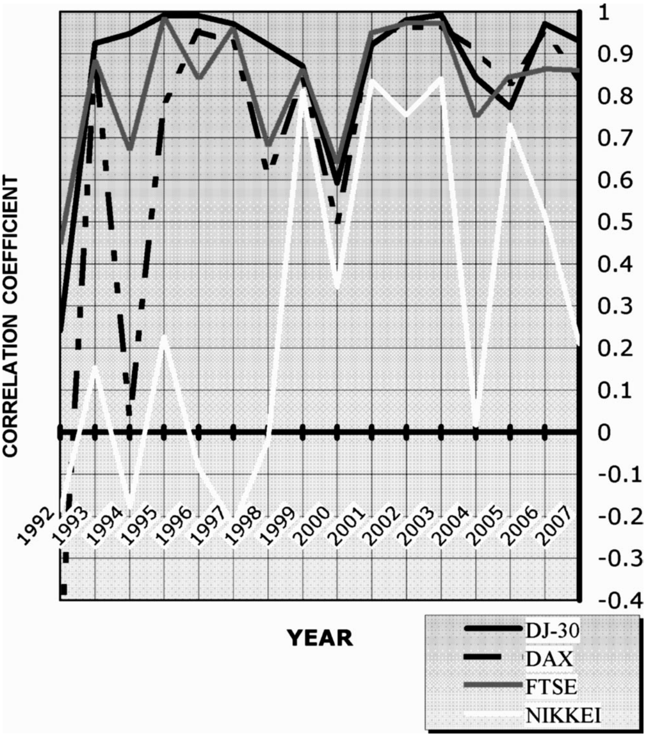


**Figure 1.4** Yearly correlation variation between the S&P 500 and leading international indices from 1992–2007. The correlation with the DJ-30 is depicted by a black line, with Germany's DAX by a dashed line, with the FTSE by a grey line, and with the Nikkei by a white line. Notice the correlation volatility, especially before 1996. The Nikkei had the weakest and most volatile correlation with the S&P 500.

There is a widely held belief that, because commodities and currencies are traded on very thin margins, they are just too risky and can lead to financial ruin. Visions of wheat being delivered to the trader's front yard, margin calls or stories of consecutive "limit down days" add fuel to the fire. Claims that "over 90 % of all futures traders lose money over time" do not help either.

Because of their low correlation to equities, most commodities are attractive diversification candidates as they can lead to a large increase in return while simultaneously reducing risk. Furthermore, futures diversification is particularly effective in declining stock markets, just where it is needed most. During periods of very low or negative stock returns, commodities (except industrial metal futures) dominate the portfolio return, acting as a hedge, or buffer, in falling markets. The benefit of including foreign stocks is not so clear as the world has gotten smaller due to the ability to communicate almost instantaneously.

Unfortunately, the approach of most novice investors or even fund managers is to have no risk management at all and it becomes obvious too late that this is an extremely dangerous omission. A fund or portfolio manager should not be evaluated only by the return he has achieved. Another important criterion of his performance is the portfolio risk exposure over time. A good benchmark of that risk is the standard deviation of returns. This is a measure of how far apart the monthly or yearly returns are scattered around the average.

Correlation is a relatively simple concept but absolutely mandatory in the use of investments. It basically refers to whether or not different investments or asset classes will move at the same time for the same reason and in the same direction. To be effective, diversification must involve asset classes that are not correlated (that is, they do not move in the same direction at the same time). High positive correlation reduces the benefits of diversification. On the other hand, selecting uncorrelated or negatively correlated asset classes not only reduces the downside volatility in the performance curve of the portfolio to a minimum but can also increase overall profitability as well. An example will help illustrate the basics of diversification.

Suppose you are considering diversifying your stock portfolio by adding an uncorrelated commodity future from the energy complex. If you invest your entire equity in either stocks or crude oil futures, and returns vary in the future as they have in the past, your equity line (in points) will be similar to the charts in the bottom window of Fig. 1.5. If, however, you invest 70 % of your initial capital in stocks and 30 % in crude oil futures, your equity line will be similar to the top chart in Fig. 1.5. You can clearly see that the portfolio's returns are not nearly as volatile as are those of the individual investments.

The reason for the reduction in volatility is that stocks did not move in the same direction at the same time with crude oil futures. Thus, a crucial factor for constructing portfolios is the degree of correlation between investment returns. Diversification provides substantial risk reduction if the components of a portfolio are uncorrelated.

In fact, it is possible to reduce the overall risk of the portfolio to almost zero if enough investment opportunities having non-correlated returns are combined together!

Maximum return, however, is also proportional to risk. Low risk investments produce low returns and speculative or riskier investments can produce higher returns. Thus reducing risk can also reduce return. Like everything else in life, the best solution is a compromise between risk and return.

The problem is therefore reduced to finding an efficient portfolio that will maximize expected return according to one's individual risk preferences. The following example will help illustrate the basics of selecting an appropriate portfolio of securities or asset classes.

Let's suppose that we want to invest in international equities but also diversify into futures and forex. For simplicity's sake I include only one sample from each asset class, for example crude oil futures to represent commodities, gold to represent


**Figure 1.5** Monthly comparison chart of the S&P 500 (bar chart with the scale on the right Y-axis) and light sweet crude oil futures continuous contract (NYMEX:CL) (line chart below) from January of 1994 to January of 2008. The equity line of a composite portfolio consisting of 70 % equities (represented by the S&P 500) and 30 % oil futures is plotted in the top window. The composite portfolio produced better returns with less volatility.

precious metals and the British pound to represent foreign exchange. International equities are represented by the S&P 500 (or a stock selection tracking the S&P), one European (the FTSE 100) and one Asian (the Hang Seng) index. In Table 1.1, I have prepared the average yearly percentage returns and standard deviation of returns. I have also calculated, at the bottom of the table, the total average 10-year return and the standard deviation of returns using Excel's STDEV function.

The correlation coefficients between the selected asset classes or indices are listed in Table 1.2. These coefficients are based on monthly percentage yields and are calculated, as part of this study, over the same 10-year period. As discussed later, the correlation coefficients play a role in selecting the asset class allocation.

You can see from Table 1.2 that the S&P 500 is correlated only with international indices. Among the other asset classes, the British pound is weakly correlated with gold ($r = 0.36$) and negatively correlated with the FTSE ($r = -0.30$) so it might be beneficial to include the pound together with the FTSE but not with gold. Gold is

**Table 1.1** All stock index returns include dividends. Also foreign index returns were converted to US dollars. The bond returns were obtained from the Lehman Brothers website (http://www.lehman.com/fi/indices/) and concerned aggregate returns of the US Bond Index. British pound (GBP/USD) returns include price appreciation versus the USD and also interest income. Oil returns were obtained from the historical continuous light sweet crude oil contract (NYMEX:CL).

| Annual returns | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | S&P 500 | FTSE | Hang Seng | Bonds | GBP | Gold | Crude oil | Cash |
| 1997 | 33.36 | 23.98 | −17.3 | 9.65 | 2.31 | −21.8 | −31.9 | 5.25 |
| 1998 | 28.58 | 17.02 | −2.15 | 8.69 | 6.47 | −0.26 | −31.7 | 5.06 |
| 1999 | 21.04 | 17.22 | 71.51 | −0.82 | 2.37 | 0.00 | 112.4 | 4.74 |
| 2000 | −9.10 | −15.6 | −8.96 | 11.63 | −1.85 | −5.55 | 4.69 | 5.95 |
| 2001 | −11.9 | −17.6 | −22.0 | 8.44 | 1.39 | 2.48 | −26.0 | 4.09 |
| 2002 | −22.1 | −8.79 | −15.1 | 10.26 | 13.56 | 24.80 | 57.3 | 1.70 |
| 2003 | 28.68 | 27.60 | 38.55 | 4.10 | 14.35 | 19.09 | 4.23 | 1.07 |
| 2004 | 10.88 | 18.43 | 16.30 | 4.34 | 12.25 | 5.58 | 33.61 | 1.24 |
| 2005 | 4.91 | 9.70 | 7.82 | 2.43 | −5.78 | 18.00 | 40.48 | 3.00 |
| 2006 | 15.79 | 27.77 | 37.35 | 4.33 | 18.85 | 23.17 | 0.02 | 4.76 |
| Mean | 10.02 | 9.97 | 10.61 | 6.31 | 6.39 | 6.55 | 16.31 | 3.69 |
| St Dev | 19.14 | 17.53 | 30.35 | 4.00 | 8.02 | 14.73 | 45.67 | 1.80 |

also very weakly correlated with crude oil ($r = 0.18$). As you will see later, however, in the case of low correlations the volatility and not the correlation coefficient is the dominant factor to consider in reducing portfolio risk. Bonds had very low correlation with the British pound ($r = 0.10$) and crude oil ($r = 0.11$) but were not particularly correlated with the others.

**Table 1.2** Pearson's correlation of monthly percentage yields for the 10-year period from 1997 to 2006.

| Correlation of monthly returns | | | | | | | |
|---|---|---|---|---|---|---|---|
| | S&P 500 | FTSE | Hang Seng | Bonds | GBP | Gold | Crude oil |
| S&P 500 | 1 | 0.81 | 0.57 | −0.06 | −0.09 | −0.04 | −0.03 |
| FTSE | 0.81 | 1 | 0.57 | −0.02 | −0.30 | −0.06 | 0.03 |
| Hang Seng | 0.57 | 0.57 | 1 | 0.05 | 0.00 | 0.11 | 0.18 |
| Bonds | −0.06 | −0.02 | 0.05 | 1 | 0.10 | 0.08 | 0.11 |
| GBP | −0.09 | −0.30 | 0.00 | 0.10 | 1 | 0.36 | 0.00 |
| Gold | −0.04 | −0.06 | 0.11 | 0.08 | 0.36 | 1 | 0.18 |
| Oil | −0.03 | 0.03 | 0.18 | 0.11 | 0.00 | 0.18 | 1 |

**Table 1.3** Risk reduction associated with asset allocation and correlations. The standard deviation of each portfolio was calculated in a separate spreadsheet by adding the annual returns of each asset class according to their percentage weights in the portfolio and then calculating the standard deviation of the annual returns for the entire 10-year period of the study.

| Portfolio allocation | US stocks only | Stocks & bonds | International stocks & bonds | Stocks, bonds & futures | Minimum risk | Maximum return |
|---|---|---|---|---|---|---|
| S&P 500 | 100 % | 70 % | 50 % | 40 % | 10 % | 60 % |
| Cash | | 10 % | 10 % | | | |
| FTSE | | | 10 % | | | |
| Hang Seng | | | 10 % | | | 10 % |
| Bonds | | 20 % | 20 % | 40 % | | |
| GBP | | | | | 55 % | |
| Gold | | | | 10 % | | |
| CRB | | | | | | |
| Oil | | | | 10 % | 15 % | 30 % |
| Average % Return | 10.0 | 8.64 | 8.69 | 8.81 | 8.92 | 11.96 |
| Standard deviation | 19.1 | 13.1 | 12.5 | 6.86 | 6.38 | 18.70 |
| Risk adj. return | 0.52 | 0.66 | 0.70 | 1.28 | 1.40 | 0.64 |

A comparison of the returns, standard deviation and risk adjusted return of the four hypothetical passive portfolios shows the real effect of diversification (Table 1.3). The first portfolio (in the third column of Table 1.3) contained a typical allocation of asset classes found in an average US fund, i.e. 70 % equities, 20 % bonds and 10 % money market. By including uncorrelated assets such as bonds and cash (with zero correlation with the S&P), a risk reduction of 31 % was achieved with only 1.4 % reduction in returns. A further 5 % risk reduction was accomplished by including international equities (10 % British and 10 % Hong Kong equities). The main reason for including international equities was to improve on US equity returns, but this was not the case in the hypothetical portfolio as the Hang Seng underperformed the S&P 500 during the 1997–1998 Asian financial crisis. Commodities, however, were the real star of the show as they played an important role in significantly reducing risk and, at the same time, increasing return.

This is evident from the standard deviation of returns of the third hypothetical portfolio. A huge 64 % reduction in risk was achieved by including gold (10 %) and crude oil futures (10 %), even though the standard deviation (and risk) of investing in crude oil alone was more than double that of the S&P.

The fourth portfolio was obtained by finding the best allocation (highest return) with the minimum risk. This produced a portfolio consisting of 30 % US equities, 55 % bonds and 15 % oil futures. The relatively high percentage allocation of bonds was to be expected as their standard deviation was the lowest of the group. The presence of the highly volatile oil futures in the minimum risk portfolio, however, was certainly a surprise.

This portfolio reduced risk by an astonishing 64 %, sacrificing only 1.2 percentage points in return compared to the equities only portfolio.

The relatively low performance of this portfolio was no surprise as the standard deviation is proportional to returns: the smaller the standard deviation, the smaller the risk and, of course, the smaller the potential magnitude of the return. There is therefore a limit beyond which the expected return cannot be increased without increasing risk.

Finally I used Excel's Solver to maximize return without increasing the risk more than the first (equities) portfolio. This portfolio (last column in Table 1.3) included 60 % US stocks, 10 % international equities and 30 % crude oil futures. It outperformed the S&P 500 by almost 2 percentage points with slightly less risk. Typically, futures can be added up to a maximum 30 % allocation while maintaining a risk advantage over a portfolio without futures.

In maximizing the return I had to constrain the risk to lower than the first portfolio, otherwise the solver produced a portfolio consisting of 100 % oil futures which is unacceptable. Similarly in minimizing risk (fourth portfolio) I had to specify a minimum return otherwise the solution also produced an unacceptable portfolio consisting mostly of cash and bonds. I also had to constrain the allocation percentages to positive values otherwise the solution occasionally included negative allocations indicating selling the asset short rather than buying.

Of course future performance rarely measures up fully to past results. While historical relations between asset classes may provide a reasonable guide, rates of return are often less predictable. In addition, as you can see from Fig. 1.4, correlations can also change over time.

One solution is to rebalance the portfolio on a set time period to take into account the most recent correlations in order to maintain the desired level of risk exposure. This method of asset allocation, is not the only one, however.

A different, dynamic rather than static, approach would involve changing asset weights depending on market conditions. This can be accomplished by reducing the allotment of equities in favor of cash, precious metals or foreign exchange in a down market. A dynamic asset allocation trading system is also discussed in Chapter 16 of this book.

# 2
# Correlation

*Study the past if you would divine the future.*

–Confucius

## 2.1 THE CORRELATION COEFFICIENT

### 2.1.1 Pearson's Correlation

The correlation coefficient measures the strength and direction of correlation or co-relation between two variables.

There are several methods, depending on the nature of data being studied, for calculating the correlation coefficient, but the best known is Pearson's product-moment correlation coefficient, usually denoted by $r$, which is obtained by dividing the covariance of the two variables by the product of their standard deviations. The formula for calculating Pearson's $r$ is:

$$r = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \tag{2.1}$$

where $\sigma_{XY}$ is the covariance of variables $X$ and $Y$ and $\sigma_{X,}$ $\sigma_y$ are their standard deviations. The covariance is the cross product of the deviations from the mean and is calculated by the following formula:

$$\sigma_{XY} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n - 1}. \tag{2.2}$$

Unfortunately the size of the covariance depends on the values of measurement of each variable and is not normalized between $-1.0$ and $+1.0$ as is the correlation coefficient.

Substituting for the covariance in (1.1) we get:

$$r = \frac{\sum z_x z_y}{\sigma_x \sigma_y (n-1)} \tag{2.3}$$

which is the average product of the $z$-scores, where $z_x = \sum(x_i - \bar{x})$ and $z_y = \sum(y_i - \bar{y})$.

The correlation coefficient varies between $-1$ and 1. A value of $+1$ indicates a perfect linear relationship with positive slope between the two variables; a value of $-1$ indicates a perfect linear relationship with negative slope between the two variables and a correlation coefficient of 0 means that there is no linear relationship between the variables. Values in between indicate the degree of correlation and their interpretation is subjective depending to some extent on the variables under consideration.

The interpretation is different for medical research, social, economic or financial time series data. In the case of financial time series the interpretation can again be different depending on whether we compare raw price data or percent changes (yields), as the direct calculation of the correlation based on absolute prices tends to overestimate the correlation coefficient as relations between financial price series are seldom linear. Correlations based on price percent changes, on the other hand, produce more realistic values for the correlation coefficient as they deviate less from linearity.

Therefore, although the correlation coefficient between two time series is unique, two different interpretations are included in Table 2.1, according to the method used for the calculation, in order to take into account the error resulting from the violation of the linearity assumption.

**Table 2.1** Interpretation of Pearson's correlation coefficient. The second column applies to the correlation between raw price data and the last column to percent weekly changes or yields. The interpretation for negative values of Pearson's correlation is exactly the same.

| Correlation coefficient $r$ | Interpretation | |
|---|---|---|
| Absolute value | Price comparison | Percent changes |
| 0.9 to 1 | Extremely strong | Extremely strong |
| 0.8 to 0.9 | Very strong | Very strong |
| 0.7 to 0.8 | Strong | Very strong |
| 0.6 to 0.7 | Moderately strong | Strong |
| 0.5 to 0.6 | Moderate | Moderately strong |
| 0.4 to 0.5 | Meaningful | Moderate |
| 0.3 to 0.4 | Low | Meaningful |
| 0.2 to 0.3 | Very low | Low |
| 0.1 to 0.2 | Very slight | Very low |
| 0 to 0.1 | Non-existent | Non-existent |

A more precise interpretation arising from the correlation coefficient is recommended by some statisticians and requires one further calculation. If the correlation coefficient is squared, the result, commonly known as $r^2$ or $r$ square or coefficient of determination (see also Section 2.1.2), will indicate approximately the percent of the "dependent" variable that is associated with the "independent" variable or the proportion of the variance in one variable associated with the variance of the other variable. For example, if we calculated that the correlation between the S&P 500 and the 10-year Treasury yield (TNX) is 0.50 then this correlation squared is 0.25, which means that 25 % of the variance of the two indices is common. In thus squaring correlations and transforming the result to percentage terms we are in a better position to evaluate a particular correlation.

There is also another factor we must consider when we try to interpret the correlation coefficient – the number of points we have used.

If we plotted only two points, then we would be bound to get a straight line between them. With three points there is still a chance that the points will lie close to a straight line, purely by chance. Clearly, a high correlation coefficient on only a few points is not very meaningful.

Traders often need to know if time series of commodity or stock prices are cyclic and, if they are, the extent of the cycle. The correlation coefficient can also be used in this case by testing for auto-correlation at different lags (testing whether values in a given series are related to other values in the same series). By doing many correlations with differing lags, the extent or duration of the cycle can be determined.

### 2.1.2 Coefficient of determination

The coefficient of determination $r^2$ is the square of Pearson's correlation coefficient. It represents the percent of the data that is the closest to the line of best fit. For example, if $r = 0.922$, then $r^2 = 0.850$, which means that 85 % of the total variation in $y$ can be explained by the linear relationship between $x$ and $y$ (as described by the regression equation). The other 15 % of the total variation in $y$ remains unexplained and stems from other exogenous factors. In regression, the coefficient of determination is useful in determining how well the regression line approximates the real data points but it can also be used (as explained above) to interpret the correlation coefficient.

### 2.1.3 Spearman's $\rho$

The best known non-parametric correlation coefficient is Spearman's rank correlation coefficient, named after Charles Spearman and often denoted by the Greek letter $\rho$ (rho). Unlike Pearson's correlation coefficient, it does not require the assumption that the variables are normally distributed but, like Pearson's correlation, linearity is still an assumption.

The main advantage of using the Spearman coefficient is that it is not sensitive to outliers because it looks at ranks as opposed to actual values.

The formula for calculating Spearman's correlation, $\rho$ is:

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)} \tag{2.4}$$

where $d$ is the difference between paired ranks on the same row.

Similar to Pearson's correlation coefficient, Spearman's $\rho$ takes values between $-1$ and $+1$. In calculating Spearman's coefficient some information is lost because the prices are converted to ranks. Therefore, when two variables appear to be normally distributed it is better to use Pearson's correlation coefficient.

The concept of correlation will be discussed extensively in the rest of the book and should be fully understood by the readers, so I have included an example of calculating both Pearson's and Spearman's correlation coefficients in the Excel worksheets in Tables 2.2 and 2.3 respectively.

## 2.2 ASSUMPTIONS

### 2.2.1 Linearity

The formula used when calculating the correlation coefficient between two variables makes the implicit assumption that a linear relationship exists between them. When this assumption is not true, the calculated value can be misleading. In practice this assumption can virtually never be confirmed; fortunately, the correlation coefficient is not greatly affected by minor deviations from linearity. However, it is always prudent to look at a scatterplot of the variables of interest before making important conclusions regarding the relation between two variables.

To understand what a linear relationship is, consider the scatterplots in Figs. 2.1, 2.2 and 2.3.

The first scatterplot in Fig. 2.1, between the S&P 500 and the British FTSE 100, illustrates an approximate linear relationship, as the points fall generally along a straight line. Keep in mind that in the financial markets there is no such thing as a perfect linear relationship. In contrast, the plot in Fig. 2.2 between the S&P 500 and the Nasdaq 100 exhibits a curvilinear relationship. In this case increasingly greater values of the Nasdaq 100 are associated with increasingly greater values of the S&P 500 up to a certain value (approximately 1200 on the Nasdaq scale).

A statistician, not knowing what each variable represents, would try to fit a cubic line of the form

$$\mathrm{SP} = \mathrm{a} + \mathrm{b.NDX} + \mathrm{c.NDX}^2 + \mathrm{d.NDX}^3$$

and predict future values of the S&P 500 using the cubic equation above.

**Table 2.2** Example of calculating Pearson's $r$ in Excel. The variables X and Y are the daily percent change in gold and the dollar index respectively from 8 December 2006 to 29 December 2006.

| A | | B | C | D | E | F | G | H | 1 |
|---|---|---|---|---|---|---|---|---|---|
| Date | | X | Y | $X - \mu$ | $(X - \mu)^2$ | $Y - \mu$ | $(Y - \mu)^2$ | $(X - \mu)(Y - \mu)$ | 2 |
| 12-8-06 | | −1.344 | 0.640 | −1.386 | 1.921 | 0.570 | 0.325 | −0.790 | 3 |
| 12-11-06 | | 0.929 | −0.132 | 0.887 | 0.787 | −0.202 | 0.041 | −0.179 | 4 |
| 12-12-06 | | −0.008 | −0.289 | −0.050 | 0.003 | −0.359 | 0.129 | 0.018 | 5 |
| 12-13-06 | | −0.325 | 0.470 | −0.367 | 0.135 | 0.400 | 0.160 | −0.147 | 6 |
| 12-14-06 | | −0.430 | 0.444 | −0.472 | 0.223 | 0.374 | 0.140 | −0.176 | 7 |
| 12-15-06 | | −1.664 | 0.406 | −1.706 | 2.910 | 0.336 | 0.113 | −0.573 | 8 |
| 12-18-06 | | 0.163 | −0.036 | 0.121 | 0.015 | −0.106 | 0.011 | −0.013 | 9 |
| 12-19-06 | | 1.088 | −0.667 | 1.046 | 1.094 | −0.737 | 0.543 | −0.771 | 10 |
| 12-20-06 | | −0.402 | 0.096 | −0.444 | 0.197 | 0.026 | 0.001 | −0.011 | 11 |
| 12-21-06 | | −0.250 | 0.072 | −0.292 | 0.085 | 0.002 | 0.000 | −0.001 | 12 |
| 12-22-06 | | 0.332 | 0.263 | 0.290 | 0.084 | 0.193 | 0.037 | 0.056 | 13 |
| 12-26-06 | | 0.613 | 0.298 | 0.571 | 0.326 | 0.228 | 0.052 | 0.130 | 14 |
| 12-27-06 | | 0.401 | −0.107 | 0.359 | 0.129 | −0.177 | 0.031 | −0.064 | 15 |
| 12-28-06 | | 1.133 | −0.179 | 1.091 | 1.190 | −0.249 | 0.062 | −0.272 | 16 |
| 12-29-06 | | 0.394 | −0.227 | 0.352 | 0.124 | −0.297 | 0.088 | −0.105 | 17 |
| | $\Sigma$ | 0.630 | 1.052 | | 9.223 | | 1.733 | −2.897 | 18 |
| | N | 15 | 15 | | | Covariance $\sigma_{xy}$ = | | −0.207 | 19 |
| mean | $\mu$ | 0.042 | 0.070 | | | Pearson's Correlation **r** | | −0.725 | 20 |
| | $\sigma$ | 0.812 | 0.352 | | | | | | 21 |
| | | | | | **Excel Formula** | | | | |
| ROW | | 17 | | | 19 | 20 | 21 | | |
| COLUMN | | | | | | | | | |
| B | | | | | COUNT(B3:B17) | B18/B19 | SQRT(E18/(B19−1)) | | |
| C | | | | | COUNT(C3:C17) | C18/C19 | SQRT(G18/(C19−1)) | | |
| D | | B17−B$20 | | | | | | | |
| E | | D17*D17 | | | | | | | |
| F | | C17−C$20 | | | | | | | |
| G | | F17*F17 | | | | | | | |
| H | | D17*F17 | | | H18/(B19−1) | H19/B21/C21 | | | |

**Table 2.3** Example of calculating Spearman's $\rho$ in Excel. The variables X and Y are the daily percent change in gold and the dollar index respectively from 8 December 2006 to 29 December 2006.

| A | B | C | D | E | F | 1 |
|---|---|---|---|---|---|---|
| Date | X | Y | Rank | Rank | $d^2$ | 2 |
| 12-8-06 | −1.344 | 0.640 | 14 | 1 | 169 | 3 |
| 12-11-06 | 0.929 | −0.132 | 3 | 11 | 64 | 4 |
| 12-12-06 | −0.008 | −0.289 | 9 | 14 | 25 | 5 |
| 12-13-06 | −0.325 | 0.470 | 11 | 2 | 81 | 6 |
| 12-14-06 | −0.430 | 0.444 | 13 | 3 | 100 | 7 |
| 12-15-06 | −1.664 | 0.406 | 15 | 4 | 121 | 8 |
| 12-18-06 | 0.163 | −0.036 | 8 | 9 | 1 | 9 |
| 12-19-06 | 1.088 | −0.667 | 2 | 15 | 169 | 10 |
| 12-20-06 | −0.402 | 0.096 | 12 | 7 | 25 | 11 |
| 12-21-06 | −0.250 | 0.072 | 10 | 8 | 4 | 12 |
| 12-22-06 | 0.332 | 0.263 | 7 | 6 | 1 | 13 |
| 12-26-06 | 0.613 | 0.298 | 4 | 5 | 1 | 14 |
| 12-27-06 | 0.401 | −0.107 | 5 | 10 | 25 | 15 |
| 12-28-06 | 1.133 | −0.179 | 1 | 12 | 121 | 16 |
| 12-29-06 | 0.394 | −0.227 | 6 | 13 | 49 | 17 |
| Σ | 0.630 | 1.052 | | | 956.00 | 18 |
| N | 15 | | | | | 19 |
| | | Spearman's Correlation $\rho =$ | | | −0.707 | 20 |
| | | **Excel Formula** | | | | |
| **ROW** | 17 | | 18 | | 20 | |
| **COLUMN** | | | | | | |
| B | | | SUM(B3:B17) | | | |
| C | | | SUM(C3:C17) | | | |
| D | RANK(B17,B$3:B$17) | | | | | |
| E | RANK(C17,C$3:C$17) | | | 1−6*F18/B19/(B19^2−1) | | |
| F | (D17−E17)^2 | | SUM(F3:F17) | | | |