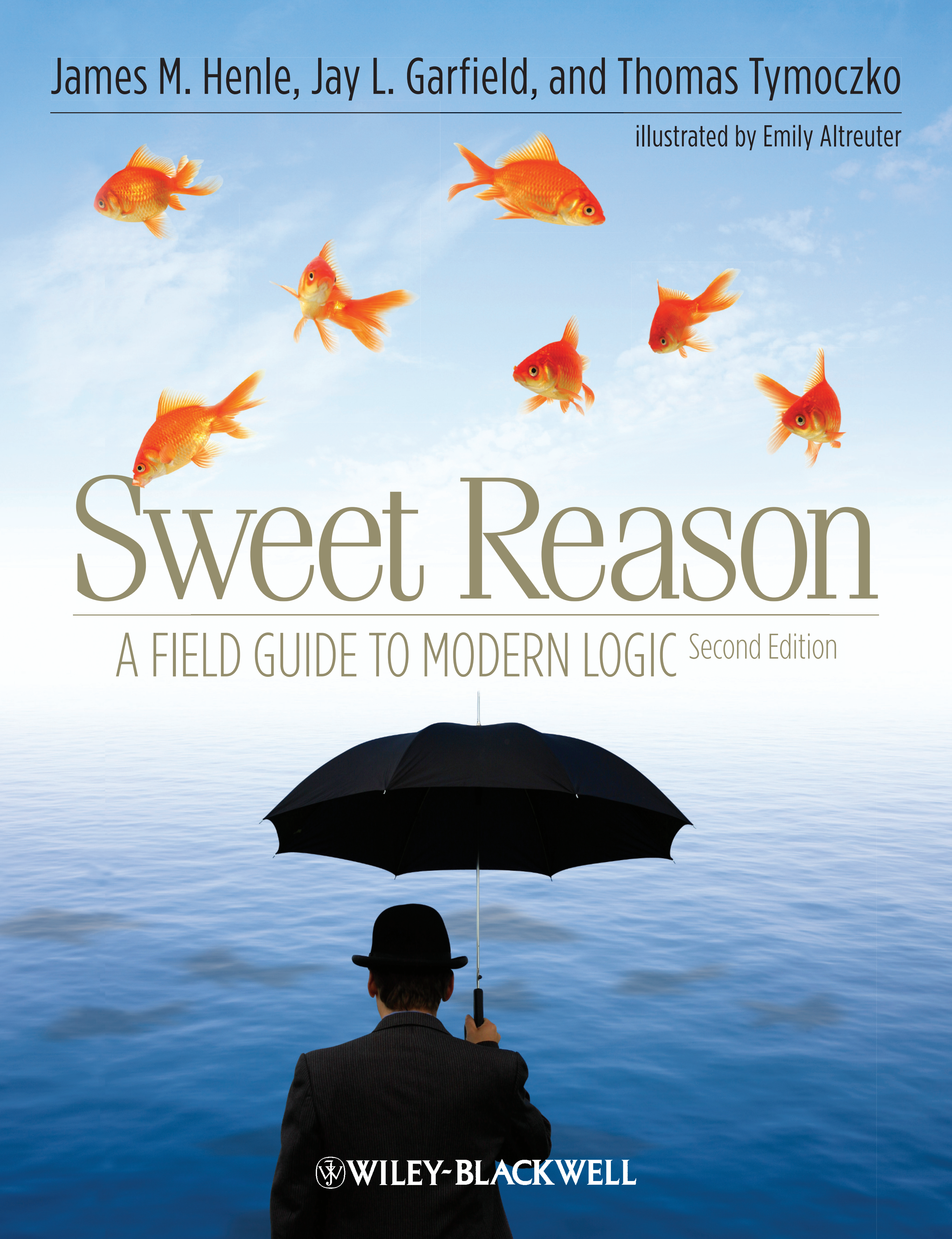

James M. Henle, Jay L. Garfield, and Thomas Tymoczko

illustrated by Emily Altreuter

Sweet Reason

A FIELD GUIDE TO MODERN LOGIC

Second Edition

WILEY-BLACKWELL

SWEET REASON

Sweet Reason pulls off the impossible: it provides a fun-to-read but also competent introduction to logic. Students in any discipline will find the text to be an intriguing first course in logical theory.

J.C. Beall, University of Connecticut and University of Otago

Introductory logic books are a dime a dozen. But this one's different. No, really. With a unique combination of philosophical nous, paradox, humor, and – often provocative – exercises, it teaches the elements of both formal logic and critical reasoning. And it shows logic as a living, breathing, evolving, stimulating, subject. If you don't want to get interested in logic, don't use this book.

Graham Priest, City University of New York Graduate Center

This extraordinary book, refined over the years in a very successful course at Smith College, is unique in scope among introductory logic texts, beginning with critical thinking, moving through a first-rate treatment of standard propositional and predicate logic, and introducing students along the way to a variety of more advanced topics, including modal logic, many-valued logics, set theory, cardinal and ordinal arithmetic, the logic of probability, and the logic of paradox.

John Horty, University of Maryland

James M. Henle, Jay L. Garfield and Thomas Tymoczko

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To those taught us Logic, Gene Kleinberg,
Nuel D Belnap, Jr and Hilary Putnam

Contents

Preface

This is an unusual introductory logic text. It teaches beginning students to understand logic not as a fixed body of knowledge or set of techniques, but as an active field of inquiry and intellectual controversy. We provide students with the tools to explore the nature of inference, the subtleties of language, and to test the bounds of rationality. This is a book designed to begin the education of logicians.

Sweet Reason goes deeper into the philosophy and applications of logic than standard texts. It is also more fun to read and more enjoyable to teach. We focus on the paradoxes at the heart of philosophical logic and the puzzles at the heart of mathematical logic. There are stories, there are entertainments, there are characters.

We present all the usual topics in first-order predicate logic. We also offer a unique and especially clean approach to analytic reading, writing and debate. The two areas, "formal" and "informal" logic, are thoroughly integrated in the text, each illustrating and informing the other.

We contextualize our presentation in the history and philosophy of logic, allowing us to introduce a variety of extensions to basic logic that take students to areas of exciting contemporary research: many-valued logic, modal logic, for example, and probability. Every chapter addresses both formal logic and critical thinking, as well as the philosophy of logic and its applications. Students learn more logic, enjoy it more and develop a deeper appreciation for logical inquiry through this integrated treatment of the discipline, and through exposure to controversy in the field.

Sweet Reason is ambitious but approachable and attainable. Novice logicians—that is, first-semester first-year students—do as well as philosophy majors and pre-law students. The mix of light and serious draws them in. The mix of formal and informal keeps them centered.

Not everything we teach fits within these covers. Our website contains a wealth of supplemental material, ranging from examples and exercises, to puzzles and curios, to extended discussions of history, philosophy, and mathematics. There are essays on religion, poetry, time travel, the tax code, and much more. Whenever a topic in the book is explored more deeply on the website, we place this logo in the margin of the text.

The website (**sweetreason2ed.com**) is constantly being updated, and will keep the volume current.

Problems of greater difficulty are specially marked:

1. (Ordinary problem)

2! (Hard problem)

3!! (Really hard problem)

4!!! (Absurdly hard problem)

This second edition of *Sweet Reason* is a wholesale revision of the first, reflecting our own (Jim's and Jay's) evolving pedagogy. We think that students and teachers alike will find it clearer and more enjoyable. We owe a lot to our late colleague Tom whose absence in this enterprise we feel keenly.

Colleagues near and far contributed much to the shape and content of this edition: Howard Adelman, Lee Bowie, Jill DeVilliers, Keith Devlin, Ruth Eberle, Lawry Finsen, Randy Frost, Michael Henle, Fred Hoffman, Murray Kiteley, Roman Kossak, Joe O'Rourke, Judy Roitman, Bob Roos, Lee Sallows, Dan Velleman, Stan Wagon, Marlene Wong, and Andrzej Zarach.

We are especially grateful for the support of students past and present, especially Gina Cooke, Kira Hylton, Marti McCausland, Cathy Weir, Theresa Huang, Julia Wu, Caroline Sluyter and all who cut their logical teeth on primitive versions of "Buffalo buffalo buffalo," "*The Digestor's Digest*," and "Obscure British Novels of 1873."

The second edition owes an incalculable debt to a talented team of student editors: Sarah Bolts, Ekaterina Eydelnaut, Caroline Fox, Emily Garvey, Penka Kovacheva, Juan Li, Sally Moen, and Katherine Peterson. Their many contributions include numerous problems, illustrations, and intelligent review.

We would like to salute here the late Jerry Lyons, our first editor and constant counsel. Perhaps there would have been a second edition, but without his encouragement and enthusiasm there wouldn't have been a first.

Tom Tymoczko died in 1995 after a short illness. He was a remarkable philosopher who made important contributions to the philosophies of mind, epistemology, language, and especially the philosophy of mathematics. His work compelled attention for a variety of reasons. He combined an appreciation for the unchanging nature of his subjects with a sharp understanding of their mutability. His insight into mathematical practice could almost be described as hip. His ideas were clear and he wrote about them with great clarity. As colleague and friend, we miss him.

Jim Henle and Jay Garfield
June 2010

How critical is Logic? I will tell you. In every corner of the known universe, you will find either the presence of logical arguments or, more significantly, the absence. (V. K. Samadar)

What Is Logic?

That's hard to say.

Logic is about relationships among statements, about the abstract structure of statements, and about the nature of arguments. A logic is an attempt to understand when one statement follows from other statements, and why. Logic is not a settled body of knowledge, but a domain of inquiry, in which we encounter different logics for different purposes, and debates among logicians about the nature of these logics and their relative merits.

We're going to show you a number of logics and introduce you to some of the challenges logic provides. You will encounter unfamiliar and sometimes perplexing ideas. You will learn a set of techniques for thinking and writing, and will gain a deeper appreciation of structure. You will think and write more clearly. You will debate more effectively.

You're also going to have a lot of fun. Some of the deepest ideas of logic appeared first as paradoxes, some of them thousands of years ago. There is a great synergy between logical puzzles and logical insight. And there is pleasure in logic. The most powerful logical ideas are also the most enchanting, the most beautiful.

So, what *is* logic?

We'll talk about that again at the end.

Sweet Reason: A Field Guide To Modern Logic, Second Edition. James M. Henle, Jay L. Garfield and Thomas Tymoczko.

Chapter One

First, a word about this chapter. Let's say you're going to learn to swim. You're 5 years old and a little afraid of the water. Your swimming teacher tells you not to be afraid, and picks you up and throws you into the pool!

You immediately start thrashing about with your arms and legs. You're really scared, but after a few seconds, you notice that you're not drowning, you're keeping your head above water. In a few more seconds, you've made your way to the side of the pool and you're hanging on to the edge trying to figure out what happened.

You didn't drown because everyone is born with swimming reflexes and instincts. When your teacher threw you in, those reflexes took command and saved you. Now that it's over, you're not as frightened of the water. You've been in the middle of the pool and survived.

This chapter is a little like that first swimming lesson. You may never have studied logic, but you do, in fact, know quite a bit. If you didn't, you could hardly speak, let alone make your way in the world.

We're going to throw everything at you. You'll be surprised at how easy it is to understand the symbols. It's easy because the logical ideas represented by the symbols are basic ideas that you've worked with all your life.

Logic can seem scary at first. If you don't know what they mean, strange symbols

$$\forall$$

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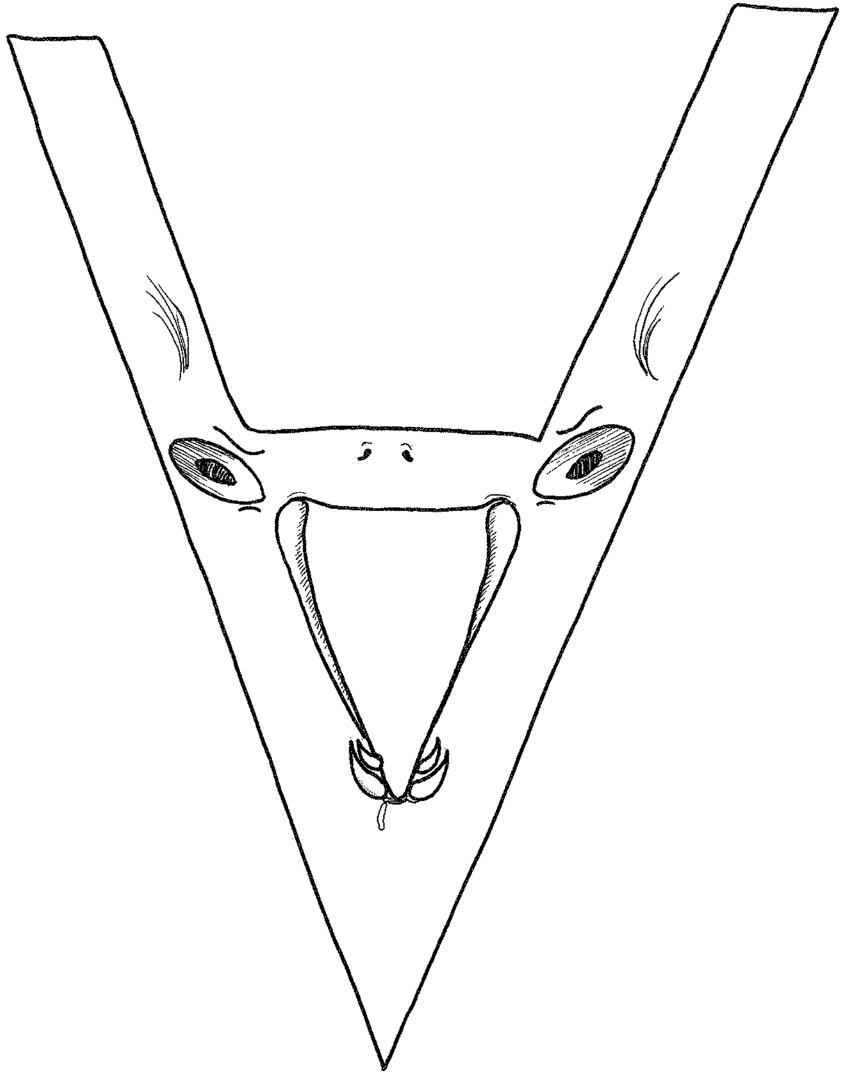
can appear frightening . . .



But don't panic. The "∀" symbol just means "everything." You'll see how it works in a moment. It's not as mean as it looks.

1.1 Introducing Formal Logic

There was only one catch and that was Catch 22, which specified that a concern for one's own safety in the face of dangers that were real and immediate was the process of a rational mind. Orr was crazy and could be grounded. All he had to do was ask; and

> *as soon as he did, he would no longer be crazy and would have to fly more missions. Orr would be crazy to fly more missions and sane if he didn't, but if he was sane he had to fly them. If he flew them he was crazy and didn't have to but if he didn't want to he was sane and had to.* (Joseph Heller, *Catch-22*)

We begin with connectives, the logical operations that link sentences to each other. We don't have many connectives; they're all familiar to you. You know them as "and", "or", "not", "if . . . then", and "if and only if". Connectives allow us to create complex statements from simple statements. Suppose A and B are statements. Then we'll use

$$A \wedge B$$

to say that both A and B are true. We'll use

$$A \vee B$$

to mean that at least one of A, B is true (A is true or B is true or both are true). We'll use

$$\neg A$$

to mean that A is *not* true. We'll use

$$A \Rightarrow B$$

to mean that if A is true then so is B. And finally we'll use

$$A \Leftrightarrow B$$

to mean that A is true if and only if B is true, that is, A and B have the same truth value.

Let's say we have these statements:

P: George is late to the meeting.
Q: The meeting is in Detroit.
R: George brings a casserole.

Example

How do we say that either George will be late or he'll bring a casserole?
Answer:

$$P \vee R$$

Example

What does $Q \Rightarrow P$ mean?
Answer: If the meeting is in Detroit then George will be late.

Example

Represent the following with symbols: The meeting is in Detroit and either George doesn't bring a casserole or George is late.
Answer: $Q \wedge (\neg R \vee P)$ Note the use of parentheses here. We'll say more about this later.

Exercises Introducing Formal Logic

Odd-numbered solutions begin on page 350

Translate the following sentences using P, Q, and R from above.

1. George is late and the meeting is in Detroit.
2. If the meeting is in Detroit, then George brings a casserole.
3. Either George is late or he does not bring a casserole.
4. George brings a casserole if and only if the meeting is in Detroit.
5. If George does not bring a casserole, he is not late.
6. If the meeting is in Detroit then George brings a casserole, and if George brings a casserole then he is late.
7. The meeting is in Detroit if and only if both George is late and he doesn't bring a casserole.
8. The meeting is in Detroit, and either George is late or he brings a casserole.

Determine the meaning of each of the following sentences.

9. $P \vee R$
10. $R \wedge \neg Q$
11. $Q \Rightarrow P$
12. $R \Leftrightarrow \neg Q$
13. $\neg P \vee (\neg Q \wedge R)$
14. $P \wedge (Q \vee R)$
15. $R \wedge (Q \Rightarrow P)$
16. $Q \vee (\neg P \Leftrightarrow R)$

The Greek philosopher Epimenides is credited with formulating a paradox that has stimulated some of the most important advances in logic from the classical period right up to yesterday afternoon (we guarantee this, no matter when you are reading these words). He, a Cretan, put it this way:

All Cretans are Liars.

Since Epimenides was a Cretan, he was asserting that he is a liar, meaning that what he says is false. So it's false that all Cretans are liars. So maybe he's not a liar. So what he is saying is true? So he is a liar! So it's false! So it's true! Paradox!

The paradox isn't perfect. Epimenides might be a liar, but some Cretans (not Epimenides) could be truth-tellers. But we can refine it.

This sentence is false.

Is it true? If so, then, since what it says is that it's false, it must be a false sentence. But then it must be true. But then it must be false! And so on.

This is the paradox of the Liar. For all its simplicity, it is very deep. Can it be resolved? In the history of logic there have been many proposals . . .

1.2 Constants and Relations

Please accept my resignation. I don't want to belong to any club that will accept me as a member. (Groucho Marx)

We can express more delicate ideas if we set up some symbols to represent individuals and other symbols to represent properties and relations. We'll use some lower case letters to refer to people.

a refers to Jim Henle (a logician)
b refers to Oprah
c refers to Tom Tymoczko (another logician)
d refers to Aristotle (a philosopher, scientist, and logician)
e refers to Hillary Clinton
f refers to Jay Garfield (yet another logician)

We'll use some upper case letters to express particular properties and relationships.

We'll use W to say that something is female. We'll write Wb to mean that Oprah is female.

We'll use G similarly to say that something is male.

We'll use M to say that two individuals are married. If we write Mdc, for example, then we are saying that Tom Tymoczko and Aristotle are married.

We'll use P to represent a relationship among three individuals. P will say that the first two individuals are the natural parents of the third. That is, if we write $Pbcd$ then we are saying that Oprahand Tom begat Ari (when you've had a little more logic, you can call Aristotle "Ari," too).

Finally, we'll use $=$ to say that two individuals are identical. If we write $e = a$ then we are saying that Hillary Clinton is Jim Henle.

Example

How can we say that both Tom and Jay are male?

Answer: $Gc \wedge Gf$.

Example

What does $Mec \Rightarrow We$ mean?

Answer: If Hillary and Tom are married to each other, then Hillary is female.

Exercises Constants and Relations

Odd-numbered solutions begin on page 350

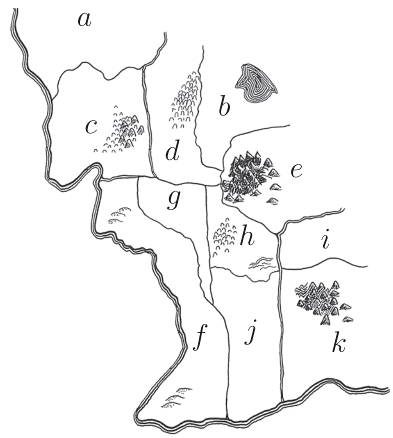
Write English sentences that express the meanings of these formulas.

1. Wc
2. Mea
3. $d = f$
4. $Pacb$
5. $Pcab$
6. $Pabc$
7. $Wa \wedge Ga$
8. $Ge \Rightarrow \neg Med$

Using only the symbols that have been introduced, write formulas that express the meanings of these sentences.

9. Hillary Clinton is married to Aristotle.
10. Aristotle is male.
11. Aristotle is married to Hillary Clinton.
12. Jim Henle is Oprah
13. Aristotle and Jay Garfield are the parents of Hillary Clinton.
14. Jim Henle is male and Tom Tymoczko is female.
15. Jay Garfield is not married to Jim Henle.
16. If Oprah and Hillary are married then Oprah is male.

The remaining problems concern the following map:



We'll use Nxy to mean that x shares a border with y at more than just a point. For example, Ngh is true because regions g and h are neighbors, but Nkh is false because k and h touch only at the corner. Furthermore, no region will be considered a neighbor of itself.

True or false?

17. Nej
18. $\neg Nah$
19. $Nkh \vee Nhe$
20. $Nbd \wedge Nbc$
21. Ngg
22. $(Ncf \wedge Njf) \wedge \neg Ncj$

23! $\neg Nij \Leftrightarrow \neg Nde$

24! $\neg Nge \Rightarrow (Nag \vee Ngh)$

"During the First World War he [Ernest Harrison] was a naval officer and shaved his mustache. On visiting Cambridge, the Master (not recognizing him) asked him at a dinner whether he was related to 'our dear Ernest Harrison.' Adopting a certain philosophical view of relations (repudiated by Russell) he replied: No."
—J. E. Littlewood, *A Mathematician's Miscellany*

1.3 Quantifiers and Variables

If you call a tail a leg, how many legs has a dog? Five? No, calling a tail a leg don't make *it a leg.* (Abraham Lincoln)

If we say, "Everyone loves ice cream," we aren't talking about anyone in particular. We're making a universal statement. We have logical notation for that. Let's say that Cx means x loves ice cream. Using the individuals of the previous section, Cb would mean that Oprahloves ice cream. Then

$$\forall x Cx$$

means "for all x, x loves ice cream." The "$\forall x$" is a way of discussing all individuals at once.

If we say, "Someone loves ice cream" we again are not talking about a particular person. We're making what we call an existential statement, a statement that something of some kind exists. There's a way to say this in our primitive logical language:

$$\exists x Cx.$$

It means "there is an x such that x loves ice cream."

The x is a variable. It doesn't stand for anyone in particular. If we use a different variable, y, the meaning is the same. Both $\forall x Cx$ and $\forall y Cy$ mean the same thing (they mean that everyone loves ice cream).

Example

How can we say that Hillary is married?
Answer: We say that there is someone who is married to Hillary , that is,

$$\exists x Mxe.$$

Equivalently, we can say $\exists x Mex$, there is someone to whom Hillary is married.

Example

What does $\forall y(Myb \Rightarrow Gy)$ mean?
Answer: It says that every y is such that if y is married to Oprah then y is male. More simply, it says that all of Oprah's spouses are male.

Exercises Quantifiers and Variables

Odd-numbered solutions begin on page 350

Translate each of the following predicate statements into English using the predicate language from the previous section (see chart below).

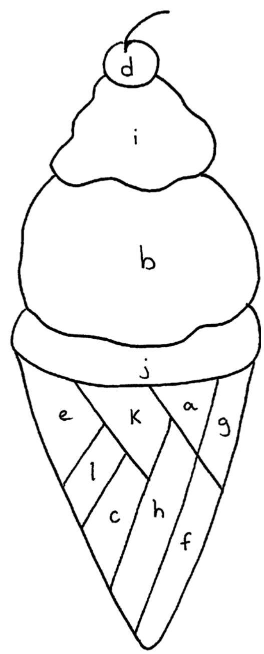
1. $\forall x Mxa$
2. $\exists y Mya$
3. $\neg\forall y Pbfy$
4. $\forall x Mbx \vee \exists y \neg Mby$
5. $\exists x(Mxd \wedge Mxb)$
6. $\forall z((z = e) \Rightarrow Wz)$
7. $\neg Gd \Rightarrow \neg\exists y Gy$
8. $\forall x(Mxa \Rightarrow Wx)$

a	Jim Henle
b	Oprah
c	Tom Tymoczko
d	Aristotle (aka Ari)
e	Hillary Clinton
f	Jay Garfield
Wx	x is female.
Gx	x is male.
Mxy	x is married to y.
$Pxyz$	x and y are the parents of z.

Translate each of the following sentences into symbolic notation.

9. Either everyone is female or everyone is male.
10. Everyone is either female or male.
11. If Tom Tymoczko is married to someone, then Tom is male.
12. Jay is a bachelor.
13. Hillary is not married to herself.
14. Jim is everyone's mother.
15. Aristotle is married to someone female, or there is a woman who is not married to Aristotle.

16! Hillary is a grandparent.



Remember that we use Nxy to mean x is a neighbor of y and that no region is next to itself. In each of the following, x stands for one of the regions in the ice cream cone above. Find x such that the statement is true.

17. Nxd
18. $Nxi \wedge Nxj$

19. $Nxf \wedge \neg Nxc$
20. $Nxe \wedge (Nxi \vee Nxh)$
21. $Nxb \wedge Nxa$
22. $Nxj \wedge Nxh \wedge \neg Nxg$
23. $Nxe \wedge \neg Nxk$
24. $Nxg \wedge \neg\exists y(Nxy \wedge Nyg)$
25. $\exists y \forall z(Nyx \wedge (Nzx \Rightarrow z = y))$
26. $\forall y(Nxy \Rightarrow Nyb)$

Have you been thinking about the paradox of the Liar? If it keeps you up at night, you have a future in logic.

One proposal to resolve the paradox is this: Perhaps the Liar sentence is neither true nor false. Maybe it has no truth-value at all, or some third, weird truth-value, like "deviant." Then, one might say, there is no paradox. The sentence is just deviant.

But consider the Strengthened Liar paradox:

This sentence is not true.

It's clear that if this sentence is true, we are once again landed into paradox, and that if it is false it's paradoxical as well. Does calling it deviant, or saying that it has no truth value, help?

No. Suppose that it has no truth value, or that it's deviant. Then it's not true, right? But that's what it says! So it *is* true! But it says that it's not! So it is! So it isn't! Back to square one.

1.4 Introducing Informal Logic

An autocrat's a ruler that does what th' people wants an' takes th' blame f'r it. A constitootional ixicutive, Hinnissy, is a ruler that does as he dam pleases an' blames th' people. (Finley Peter Dunne)

You're a first year student. You arrived two weeks ago at Sophist College, the ivy-draped liberal arts institution you dreamed of for years. Two weeks, but you're still floating on air. The academic atmosphere . . . the intellectual giants who are your professors . . . the imposing architecture . . . the excitement of campus life . . . the opportunities you see ahead . . . the challenge of the courses you've just begun . . . everything is as new and as thrilling as you had hoped.

Above all, you're in awe of the older students. They're so confident, so accomplished, so wise, so *cynical.* Well, I suppose there's nothing great about being cynical, except that you have to know a lot to be cynical, don't you? In any case, you relish those bull sessions that last until three in the morning . . . that's where it's at, that's where the world really unfolds, that's where . . .

But then one night the whole wonderful picture collapses. The discussion is about China. You just read that morning about the tight rein the government keeps on people. All you say is, "What they need is some democracy. If they would only let the people rule," and then Cathy jumps on you. Cathy, the junior you admired for her quickness, her assurance – and she seemed to like you.

"What's so terrific about democracy?" she asks. "In a democracy, the people choose, but they make terrible choices. They get freedom in the Balkans and the first thing they do is start shooting at each other. They get the vote in Iraq and they have a civil war.

"We have democracy, right? Well how great is that? We don't protect the environment, our schools are rotten, and we're in debt up to our eyeballs. If democracy is so wonderful, how come only 23 percent of the people vote here?"

You try to cut in. "But democracy has made us the most powerful, the most envied —" But she runs right over you!

"Oh, brother. We're powerful and envied because we're rich, not because of our campaign commercials. And all we do is abuse that power. And anyhow, we don't really have democracy. You know about Washington, D.C.? One of the biggest cities in the country, and they don't have self-government or representation in Congress. Why? Because it's a black city and we're all racists.

"Look at all the democracies in South America: all bankrupt. The only country down there with its act together is Chile, and it took a dictator, Pinochet, to put it on the road to recovery. You know what H.L. Mencken said? He called democracy the form of government that believes that the people know what they want and they deserve to get it – good and hard!"

You're devastated. Your deepest beliefs are in ruins! You can't say a thing because . . . well . . . everything she's saying sort of makes sense. But you still believe in democracy! You know it's right! But then, what's wrong with her arguments? What do you say?

You need to know how to argue!

There are good reasons for learning the art of argument.

First of all, you want to be able to defend your point of view. You want to persuade others. This is certainly true if you're right. And maybe it's useful even if you're wrong.

Secondly, and more nobly, you want to find out what is actually true. There is, perhaps, no better way to get to the bottom of things than to argue. When two skilled debaters engage, the best argument prevails. More often than not the winner is the truth.

Finally, the ability to argue represents power. If you can marshall your thoughts, arrange them in a logical order, and explain them clearly, people will pay attention. If your arguments are understandable and persuasive, you will be influential. *Your* issues, *your* perspectives, *your* proposals will take center stage.

In this book we'll teach you how to argue. We'll do it in stages. We'll start by showing you how to take apart an argument such as Cathy's, diagram it, and attack it. Then we'll show you how to construct your own argument, diagram it, and write it.

* * * * * * * * * * * * *

A word about Cathy. She's sort of unpleasant. Unfortunately, she appears throughout this book; she insisted on it.

But responding to her is a good logical exercise. What's her point, anyway? We'll come back to this, but first we'll think more generally about the task of identifying conclusions.

1.5 Conclusions

> *Joe DiMaggio might have hit in 56 consecutive games, a seemingly unrivaled record, but he never won 33,277 arguments in a row, like Ted Williams, the undisputed champion of contentiousness.* (David Halberstam, *The Teammates*)

The first step in tackling an argument is identifying the conclusion. This is more difficult than it sounds. You would think that anyone going to the trouble of making an argument would make sure we got the point. But that isn't always the case.

Writing is difficult. Writing arguments is especially difficult (as you will soon see). It's not surprising that it's often done poorly. That makes reading arguments a challenge. The key, and it is the key in formal logic too, is language. Unfortunately, while it is easy to say, "I would like to argue that . . ." or "My conclusion is . . ." that is too simple for most writers.

Consider the following three letters to the editor of *The New York Times*, May 11, 2005, responding to a column by Thomas Friedman arguing for an economic boycott of Iran and North Korea if they don't terminate their nuclear programs:

> *It is disturbing that Thomas L. Friedman seems to suggest that the world's most powerful countries (or groups of countries) should simply starve their opponents into submission.*
>
> *First, it would be a blatant violation of international human rights principles. Second, such measures would mostly harm those people (civilians) who have the least power to do anything about the situation in their respective countries.*
>
> *Surely Mr. Friedman does not believe that the leaders of Iran and North Korea are incapable of securing the necessities of life for themselves and their own families, and they have already demonstrated that they care little for the rest of their populations.*
>
> Jessica Crutcher

This is pretty simple. The writer is opposed to a boycott. But note that this conclusion is not explicitly stated. We have to figure that out from the list of negative effects of a boycott.

> *If China pressured North Korea to cease its weapons program by saying to Kim Jong Il, "You will shut down your nuclear weapons program and put all your reactors under international inspection, or we will turn off your lights, cut off your heat and put your*

whole country on a diet," perhaps the United States should insist that China do just that, lest we stop all our imports and bring its production machine to a grinding halt.

Lisa Calef

This letter is clearly in favor a boycott, though again it is not stated as such; instead the writer urges that the United States boycott China if China doesn't boycott North Korea.

Thomas L. Friedman is correct: there is a lot more that China and the European Union could do to deter both North Korea and Iran in their nuclear ambitions. But let us not underestimate the main attraction of obtaining such weapons: your enemies will think twice about attacking you.

Terry Phelps

This third letter is a little puzzling. What exactly is the conclusion? Should we boycott the countries? Would that address their motivation?

And what do you suppose is Cathy's conclusion in the previous section? She starts out attacking democracy. But then she complains that we don't have democracy and seems to think that's bad. Then she goes back to slamming democracy. This is one of the reasons Cathy is so hard to deal with – she jumps from one attack to another.

The best answer is that Cathy is arguing that democracy is not a good form of government. We'll begin rebutting arguments, starting with this one, in Chapter Three.

Exercises Conclusions

Odd-numbered solutions begin on page 350

The conclusion can appear anywhere in the argument, or nowhere. A good place to look for it, though, is at the beginning and at the end. A well-written argument is likely to state it in both places. Look for key words, "therefore", "so", "hence", and "consequently."

Find the conclusions of the following arguments.

1. If we have the picnic on Sunday, David can't make it. We have to have it before exam period starts on Tuesday. The later the picnic is the better, so let's make it Monday.
2. I think the solution is to raise the tax on gasoline. If gas were more expensive, people would conserve. That would reduce emissions. And the government would collect money that could be used to clean up oil spills.
3. Doug is a dog only if he plays fetch. Doug is a cat. If Doug is a cat, then he's not a dog. So Doug does not play fetch.
4. There is no real difference between classical and popular music, and it is easy to see why. Everybody agrees that jazz is popular music, but it is also classical. After all, classical music is the music that represents the highest and most distinctive music produced by a culture, the music that endures and is passed from generation to generation, and in the performance and

composition of which virtuosity is demonstrated. But jazz plays this role in African-American culture. So jazz is classical music. Therefore, since it is also popular music, there is no real difference.

5. Should we legalize marijuana? Should we make it easier for people to poison themselves? Should we provide amnesty for drug-dealers? Should we give society's blessing to a degenerate, degrading practice?
6. Should we keep drug use illegal? Should we use the army and navy to attack drug dealers? Should we glamorize a destructive habit? Should we jack up the price of drugs so that addicts kill to get high? Should we enrich South American drug-dealing terrorists?
7. The economy is crashing right now because of oil prices. The cost of gasoline is at a historic high. So raising the tax on gas would be a big mistake. It would make it impossible for small businesses to operate.
8. Censorship of speech is never justified. Speech itself never harms anybody; at most the actions inspired by it cause harm, and they can be prohibited. If speech is censored, valuable ideas will be lost to the public and individuals will be prevented from expressing their own ideas and values. Now, pornography is a kind of speech. Consequently pornography should never be censored. Now, some people might be offended by pornography, but their own emotional reaction is their problem, and should not count against the rights of others.

1.6 Dialects of Logic

Histories make men wise; poets, witty; the mathematics, subtle; natural philosophy, deep; moral philosophy, grave; logic and rhetoric, able to contend. (Francis Bacon)

Each chapter of this book will begin with sections on formal logic, followed by sections on informal logic. Each chapter will end with a section on one of the many different logics, formal and informal, that are part of the history of logic and part of current research in logic.

A Typical Chapter
Some formal logic
Some related informal logic
A logic variant

In this first chapter, the logic variant is quite tame. We thought we'd tell you about some alternate notation for the basic connectives – notation which we *won't* use but which other writers may and which you might encounter elsewhere. Knowing that the odd symbols are just alternate notation for the same ideas will help you avoid confusion. It will also help to keep you aware of the difference between symbols and what symbols stand for.

And

Many logicians, especially philosophical logicians, use & instead of $\wedge$. Indeed, the first edition of *Sweet Reason* used this symbol. Other logicians have used the letter K, a single dot · , $\cap$, u, or have simply written "*P* and *Q*" as *PQ*.

Or

There is unanimity today for the wedge, "$\vee$" Still, in the history of logic, $\cup$, $+$, A, and even $\times$ have been used for "or".

Not

It is quite common to use $\sim$ for not. Other notations include - , N, $\neg$, and placing a line or a $\sim$ above the statement letter.

If . . . then

You will see $\supset$ in many logic books. You will also see differently shaped arrows, $\rightarrow$, $--$ >, =>. In the distant past, C, and Ↄ have also been used.

If and only if

The symbol, $\equiv$, is frequently used in place of $\Leftrightarrow$. In the past, $\leftrightarrow$, $\sim$, E, and $\supset\subset$ have been used.

"If and only if" is often abbreviated **iff**. This is so handy we'll use it too. When you see "iff" it will always mean "if and only if."

That's all for now. You'll see some of these symbols in different contexts later in this book, sometimes to explain, sometimes to entertain, and in one case, to tease.

Quiz

To test your aptitude for studying logic

For each of the statements below, answer either true or false:

1. My answer to statement 2 is different from my answer to this statement.
2. My answer to statement 3 is the same as my answer to this statement.
3. Wow! This book is off to an amazing start! What a great read! These guys Jim, Jay, and Tom are AWESOME! I'll bet this wins a Pulitzer or a Nobel or an Oscar, or whatever it is they give to obscure texts in logic! I can't wait to find out what happens in the next chapter! I want to sit here and read the whole thing right now! Wow!

You may grade the quiz yourself. After you have completed writing your answers, ask yourself whether each answer is correct. For example, suppose you answer:

1. T
2. F
3. F

then the answer to statement 1 is correct (because your answer to 2 is different from your answer to 1). But your answer to 2 is incorrect (your answer to 3 is the same as your answer to 2 but you wrote 'F'). *Your own judgment is perfectly acceptable in deciding whether you have answered statement 3 correctly.*

It is possible to get a perfect score on this quiz.

Chapter Two

Sweet Reason focuses on two areas of logic: formal logic and what we are calling informal logic. The first deals with logic in the context of formal language, where statements are abstract and are often without determinate meaning.

$$P \wedge Q \qquad \forall x(Ax \vee Bx) \qquad (J \Rightarrow K) \Rightarrow (M \Rightarrow N) \qquad C \Leftrightarrow \exists y \exists z Hyz$$

The second deals with logic in the context of natural language, in this book, English. Here, statements are more often concrete and meaningful.

Today is Tuesday.   My dog has fleas.   Life is really, really strange.

There are good reasons to study formal logic. The very abstractness of formal language allows us to see logical issues clearly. Natural language is full of ambiguity and vagueness. Formal languages streamline, clarify, and simplify.

There is also good reason to study informal logic. Natural languages, like English, after all, are important human tools. We use them to understand the world, to communicate our understanding, to interact, to influence people, and to have an impact on events. Logic can help us to do all of this more effectively, and can help us to understand these aspects of our lives.

The two areas of logic are different but they're intimately connected. Formal language is abstracted from natural language. The choices made, for instance, in the definitions of $\vee$, $\wedge$, $\Rightarrow$ are based on the meaning of words in English. The world of people and events and the English language are reality checks on formal logic.

Formal logic, on the other hand, reveals the meaning and structure in natural language that words often obscure. It reveals the abstract skeletons of arguments and statements that enable them to be meaningful in the first place.

Nowhere is this connection between the formal and the informal clearer than in the question of inference. That is the focus of this chapter.

Sweet Reason: A Field Guide To Modern Logic, Second Edition. James M. Henle, Jay L. Garfield and Thomas Tymoczko.

2.1 Formal Inference

Why is this thus? What is the reason for this thusness? (Artemus Ward)

Logic is about what follows from what; it's about how to construct and understand arguments, about the relation between language and the world, and about how to tell a good argument from a bad one. We'll get to all of this, but let's begin by introducing a few terms. Some of these are technical terms in logic, and have meanings that are different from those they have in ordinary speech.

An argument is **valid** if and only if it's impossible for its premises to be true and its conclusion false. Another way to put this is to say that an argument is valid iff[1] the truth of the premises guarantee the truth of the conclusion. The premises are the reasons given in support of a conclusion. The conclusion is what we derive from the premises.

Example

$$\begin{array}{l} \quad P \\ \quad Q \\ \hline \therefore P \wedge Q \end{array}$$

The premises of the argument are the statements above the line, P and Q; the conclusion is the statement below the line, $P \wedge Q$. The dots, $\therefore$, mean "therefore".

Is it possible for the premises of this argument to be true and the conclusion false? Absolutely not. If P and Q are true, so is $P \wedge Q$. This is a valid argument.

Example

$$\begin{array}{l} \quad P \vee Q \\ \hline \therefore P \end{array}$$

Is it possible for the premises of this argument to be true and the conclusion false? Indeed it is. In our formal language P and Q are independent statements. It's possible for P to be false and Q true. In that case, the premise, $P \vee Q$, is true while the conclusion, P, is false. This is an invalid argument.

The case where P is false and Q is true is a **counterexample**. It's an example of a situation where the premises are true and the conclusion is false. A counterexample shows conclusively that an argument is invalid.

Example

$$\begin{array}{l} \quad P \\ \quad P \Rightarrow Q \\ \hline \therefore Q \end{array}$$

[1] Remember, this is "if and only if" (p.16).

Is it possible for the premises of this argument to be true and the conclusion false? Let's see. If the premises are true, then P is true. If the conclusion is false, then Q is false. But if P is true and Q is false, then $P \Rightarrow Q$ is false; $P \Rightarrow Q$ promises that if P is true then Q will be true. Thus it's not possible for the premises to be true and the conclusion false. This is a valid argument. It's so fundamental it has a name, *modus ponens*.

Example

$$\begin{array}{l} \quad \exists x Px \\ \quad \exists x Qx \\ \hline \therefore \exists x(Px \wedge Qx) \end{array}$$

Is it possible for the premises to be true and the conclusion false? The first premise says that something has property P. The second one says that something has property Q. The conclusion says that something has both property P and property Q.

This is a nice example where the real world and English can be helpful. Think of possible meanings for P and Q. Suppose Px means that x is human and that Qx means that x is a tree. Then on the planet Earth, $\exists x Px$ is true; there is a human. Also, $\exists x Qx$ is true; there is a tree. But the conclusion, $\exists x(Px \wedge Qx)$ is false. Nothing on Earth is both a human and a tree. This argument is invalid.

Again, giving the particular meanings to Px and Qx creates a counterexample which shows that the argument is invalid.

Exercises Formal Inference

Odd-numbered solutions begin on page 351

Decide, for each argument below, whether it is valid or invalid. For any invalid argument, find a counterexample.

1. $$\begin{array}{l} \quad P \vee Q \\ \hline \therefore P \end{array}$$

2. $$\begin{array}{l} \quad P \Leftrightarrow Q \\ \hline \therefore P \end{array}$$

3. $$\begin{array}{l} \quad P \wedge Q \\ \hline \therefore P \vee Q \end{array}$$

4. $$\begin{array}{l} \quad \neg\neg P \\ \hline \therefore P \end{array}$$

5. $$\begin{array}{l} \quad P \Rightarrow Q \\ \hline \therefore P \end{array}$$

6. $$\begin{array}{l} \quad P \Rightarrow Q \\ \hline \therefore Q \end{array}$$

7. $$\begin{array}{l} \quad \neg(P \Leftrightarrow Q) \\ \quad Q \\ \hline \therefore P \end{array}$$

8. $$\begin{array}{l} \quad P \Rightarrow Q \\ \hline \therefore Q \Rightarrow P \end{array}$$

9. $$\begin{array}{l} \quad \forall x Px \\ \hline \therefore Pa \end{array}$$

10. $$\begin{array}{l} \quad \forall x(Px \wedge Qx) \\ \hline \therefore Qb \end{array}$$

11. $$\begin{array}{l} \quad \exists x Px \\ \hline \therefore Pa \end{array}$$

12. $$\begin{array}{l} \quad \exists x(Px \wedge Qx) \\ \hline \therefore Qb \end{array}$$

Let's call an expression *autological* if it applies truly to itself, and *heterological* if it does not. The word, "short," for instance, applies truly to "short" ("short" is a short word) and so "short" is autological. "Long", however, is not long, and so "long" is heterological. "English" is autological because "English" is English, but "French" is heterological, because "French" is not French. It's easy to see that every expression is either heterological or autological, and that none can be both. Now here's the question: Is "heterological" heterological?

If "heterological" is heterological, then clearly it applies truly to itself. Thus, "heterological" is autological and not heterlogical. On the other hand, if "heterological" is autological, i.e., not heterological, then it doesn't apply truly to itself and so it's heterological and not autological.

This paradox is due to Kurt Grelling and Leonard Nelson. "Heterological" is heterological if and only if it isn't! What's going on here?

2.2 Informal Inference

The purpose of writing is to inflate weak ideas, obscure pure reasoning, and inhibit clarity. With a little practice, writing can be an intimidating and impenetrable fog! (Calvin (Bill Watterson))

We have a definition of validity for formal logic. It's important, so we'll repeat it.

An argument is

Valid	Iff the conclusion is true whenever the premises are true.
Invalid	Iff it's possible for the premises to be true and the conclusion false.

Arguments in English are more difficult to evaluate. We can do this most successfully when we can identify the formal argument inside. Consider this argument:

All dogs are mammals.
McLeod is a dog. (This is true.)
∴ McLeod is a mammal.

This seems pretty reasonable. The fact that all dogs are mammals and that McLeod is a dog guarantees that McLeod is a mammal. It can't be otherwise if those premises are true. Now consider this argument:

All dogs are human.
McLeod is a dog.
∴ McLeod is human.

Strange! The first premise is false! And yet, if all dogs really were human and if McLeod was a dog, wouldn't he be human? Indeed, the form of this argument is the same as the form of the first argument. We might write it as:

All *A* are *B*.
s is an *A*.
∴ *s* is a *B*.

This is the underlying form. The only differences between the two arguments are the meanings of *A*, *B*, and *s*. And the form is valid. It is impossible, no matter what meanings we attach to *A*, *B*, and *s* to make the premises true and the conclusion false. Notice that in the first argument the premises and the conclusion are true. In the second, not all the premises are true and the conclusion is false. But this doesn't violate the definition of validity which says that *if* the premises are true the conclusion must be true. There is no requirement if not all the premises are true.

All dogs are human.
Jay is a dog. (False.)
∴ Jay is human.

This is a third argument with the same underlying form. Note that here too the conditions of validity are satisfied.

Now we'll give you something invalid.

Some dogs are mammals.
McLeod is a dog.
∴ McLeod is a mammal.

To see why this is invalid, look at the underlying form.

Some *P* are *Q*.
d is a *P*.
∴ *d* is a *Q*.

Can we find a counterexample? Can we make, by a clever choice of meanings for *P*, *Q* and *d*, the premises true and the conclusion false? We can.

Some humans are female.
Jim Henle is a human. (True.)
∴ Jim Henle is female. (False.)

This argument form has sold a lot of snake oil.

Many hard-working people who tried Jay's snake oil found it changed their lives!
You're a hard-working person!
———
∴ This snake oil will change your life!

Now here's another invalid argument:

Everyone who voted Republican wore red.
My mom wore red.
———
∴ She voted Republican.

This argument form is especially common and pernicious. It's called **affirming the consequent**. Here's a counterexample:

All birds have wings.
A Boeing 747 has wings.
———
∴ A Boeing 747 is a bird.

Enough said? No argument of this form should ever convince you of anything. Again, it's a common advertising trick:

If you're hip, you wear Calvins.
You wear Calvins.
———
∴ Hey, you are SO hip!

Here's the point in this discussion: If an argument form is invalid, you can show that by coming up with a counterexample. You haven't yet learned to show that an argument is *valid*, but that will come.

One last bit of terminology. Obviously, while we care about argument form, truth is nice, too. So, we have a special name for valid arguments all of whose premises are true. These are **sound** arguments. It follows that the conclusion of a sound argument is always true.

An argument is

Valid	Iff the conclusion is true whenever the premises are true.
Invalid	Iff it's possible for the premises to be true and the conclusion false.
Sound	Iff it's valid and the premises are true.

Exercises Informal Inference

Odd-numbered solutions begin on page 351

For each argument, decide whether it is valid or invalid. If it is invalid, find a counterexample.

1.
All pies are delicious.
Rhubarb pie is a pie.
∴ Rhubarb pie is delicious.

2.
Pope Benedict XVI is pope.
The pope is infallible.
If one is infallible then everything one says is true.
∴ Everything Pope Benedict XVI says is true.

3.
If Georgia goes to college she learns logic.
Georgia does not go to college.
∴ Georgia does not learn logic.

4.
All college graduates are powerful and successful.
Hillary Clinton is powerful and successful.
∴ Hillary Clinton is a college graduate.

5.
Everyone loves logic.
If Jim loves logic, he is not bald.
∴ Jim is not bald.

6.
If Leroy is a flippet then he minks.
∴ Leroy minks.

7.
Edward chirs if and only if he does not wix.
Edward does not wix.
∴ Edward chirs.

8.
If Oprah goes to the ball then if the prince is there she will dance with him.
Oprah goes to the ball and the prince is there.
∴ Oprah dances with the prince.

9.
We go to war or we have peace.
We do not have peace.
If we send an ambassador then we do not go to war.
∴ We don't send an ambassador.

10.
You go abroad if you have a high GPA.
∴ You go abroad only if you have a high GPA.

11.
If everyone reads the book, then everyone passes the course.
If everyone passes the course, the teacher is happy.
If the teacher is happy, she brings us cupcakes.
John doesn't read the book.
∴ The teacher doesn't bring us cupcakes.

12.
If you do drugs, you drop out of school.
If you drop out of school then you have to work in McDonald's until you die.
Alberta works at McDonald's until she dies.
∴ Alberta does drugs.

There are only a finite number of English words (about 500 000). Consequently there are only a finite number of grammatical phrases in English using fewer than twenty words. But there are infinitely many natural numbers (1, 2, 3, . . .). Many phrases in English of fewer than twenty words describe natural numbers, such as "the sum of three and four," or "the highest number to which Jay ever counted plus the highest number to which Jim ever counted," or "the square root of one trillion." But there are only finitely many of these phrases. So there must be many numbers which are not described by any phrase with fewer than twenty words.

Now consider the smallest number not describable by an English phrase of fewer than twenty words. That's a number. And we just described it with an English phrase of fewer than twenty words!

This paradox was invented by G. G. Berry, a friend of Bertrand Russell.

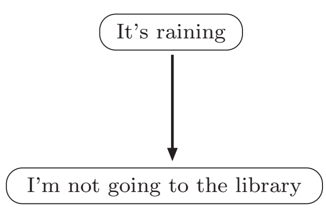
2.3 Diagramming Arguments

All generalizations are dangerous, even this one. (Alexandre Dumas)

If an argument is well-written and if we have read it correctly, then we should have a picture in our mind of the logic of the argument. We're going to represent the logic of arguments with diagrams.

It's raining so I'm not going to the library.

The conclusion is that I'm not going to the library. The reason is that it's raining. We'll diagram this as follows:



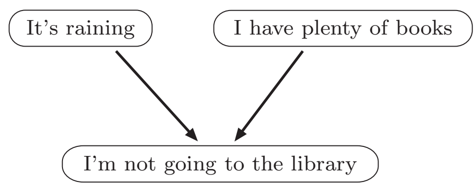
We place the conclusion at the bottom. In general, if statement $\mathcal{A}$ supports statement $\mathcal{B}$, we'll put $\mathcal{A}$ above $\mathcal{B}$ and draw an arrow from $\mathcal{A}$ to $\mathcal{B}$.

We aren't talking about valid arguments here, or even good arguments. We're simply diagramming what we think the author intended. In the next two chapters we'll start attacking arguments. When we do, we'll use what we've learned about validity.

Now here's a different argument:

> *It's raining and I have plenty of books to read so I'm not going to the library.*

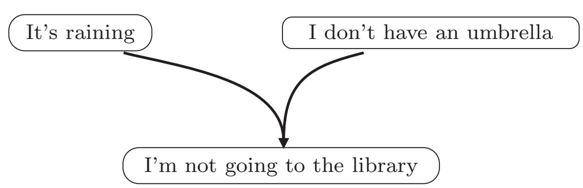
There are two reasons for not going to the library. The reasons are independent; neither one depends on the other. That is, either reason would be a reason on its own, even without the other. We diagram it like this:



Now a third argument.

> *It's raining and I don't have an umbrella so I'm not going to the library.*

This time the reasons aren't independent, they work together. Neither by itself is a reason for not going to the library – if it were raining and I did have an umbrella, I'd go. And if it weren't raining, I'd go. But taken together – rain and no umbrella – they provide a reason. We diagram the argument this way:



Here's a more complicated argument:

> *Nancy will make a terrific chair of the entertainment committee. She spent a year on the housing committee so she knows all the administrative officers. She has great people skills. And she just got a new laptop.*

Let's make a list of the statements:

a. Nancy will make a terrific chair.
b. Nancy spent a year on housing.
c. Nancy knows all the administrative officers.
d. Nancy has great people skills.
e. Nancy has a new laptop.